

Midterm Examination for MATH400-201, 2016-2017

Total: 100 points

Answer All Questions. Show All Steps.

Date: October 28th, 2016

1. (15pts) Find the general solutions to the following first order PDE:

$$yu_x + 2xyu_y = 2xu, -\infty < x < +\infty$$

2. (30pts) Consider the traffic flow problem

$$u_t + (2 - u)u_x = 0, t > 0$$

Solve for  $u(x, t)$  with the following initial conditions

$$u(x, 0) = \begin{cases} 0, -\infty < x < 0 \\ 1, 0 < x < +\infty \end{cases}$$

3. (30pts) Solve the following second order PDE:

$$u_{tt} = u_{tx}, x > 0, t > 0$$

$$u(x, 0) = 0, u_t(x, 0) = 0, x > 0$$

$$u(0, t) = t, t > 0$$

4. (10pts) Solve the following wave equation

$$\begin{cases} u_{tt} = c^2 u_{xx} + \cos(x), -\infty < x < +\infty \\ u(x, 0) = 0, u_t(x, 0) = 0 \end{cases}$$

5. (15pts) Solve the following diffusion equation

$$\begin{cases} u_t = \frac{1}{4} u_{xx}, -\infty < x < +\infty, t > 0 \\ u(x, 0) = e^{-x^2}, -\infty < x < +\infty \end{cases}$$

Note:  $k = \frac{1}{4}$  and

$$S(x, t) = \frac{1}{\sqrt{4k\pi t}} e^{-\frac{x^2}{4kt}} = \frac{1}{\sqrt{\pi t}} e^{-\frac{x^2}{t}}$$