MATH256-201/202 Homework Assignment 5 (Due Date: April 9, 2018)

Note to Section 201 students: Homework is admitted until 1pm on April 9, 2018. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. Find solutions to the following two-point boundary value problem:

$$(a)\quad y^{''}+2y=\sin(x), y(0)=0, y(\pi)=0$$

$$(b)\quad y^{''}+y=\sin(x), y(0)=0, y(\pi)=0,$$

$$(c)\quad y^{''}+y=\sin(x)-\frac{1}{2}, y(0)=0, y(\pi)=0,$$

$$(d)\quad y^{''}+y=\cos(x), y(0)=0, y(\pi)=0.$$

2. Find the Fourier series for the following function

$$(a)\ f(x)=\begin{cases} x, -\pi\le x<0; \\ 0, 0\le x<\pi; \end{cases} \qquad f(x+2\pi)=f(x)$$

$$(b)\ f(x)=\begin{cases} x^2, -1\le x<0; \\ 1, 0\le x<1; \end{cases} \qquad f(x+2)=f(x);$$

$$(c)\ f(x)=\begin{cases} x, -2\le x<0; \\ x+1, 0\le x<2; \end{cases} \qquad f(x+4)=f(x).$$

3. The following functions are given on an interval of length 1. (a) Extend evenly to be 2 periodic function and find the Fourier series; (b) extend oddly to be 2 periodic function and find the Fourier series

$$(1)\ f(x)=\sin(\pi x), 0<x<1;\quad (2)\ f(x)=\cos(\pi x), 0<x<1;\quad (3)\ f(x)=x+1, 0<x<1$$

4.Consider the following function

$$f(x)=\begin{cases} x, & -1\le t<0 \\ 1, & 0\le x<1 \end{cases} \qquad f(x+2)=f(x)$$

(a) Compute the first three coefficients of full Fourier series expansion a_0, a_1, b_1.

(b) Find out the values of the full Fourier series expansion at $x = -\frac{1}{2},\ 0,\ \frac{1}{2}$. Hint: you can use the following theorem:

$$\frac{a_0}{2} + \sum_{j=1}^{\infty}(a_n \cos(\frac{\pi x}{L}) + b_n \sin(\frac{\pi x}{L})) = \frac{1}{2}(f(x-) + f(x+))$$

where f is a piecewise smooth function.

5. Use the method of separation of variables to solve the following heat equation

$$u_t = 2u_{xx}, \quad 0 < x < \frac{\pi}{2},\ t > 0;$$

$$u(x,0) = \sin(2x) + 4\sin(8x), \quad 0 < x < \frac{\pi}{2};$$

$$u(0,t) = 0, \quad u(\frac{\pi}{2}, t) = 0,\ t > 0.$$

6. Use the method of separation of variables to solve the following heat equation

$$u_t = 2u_{xx}, \quad 0 < x < \pi,\ t > 0;$$

$$u(x,0) = 0, \quad 0 < x < \pi;$$

$$u(0,t) = 1, \quad u(\pi,t) = -1,\ t > 0.$$

What is the asymptotic behavior of $u(x,t)$ as $t \to +\infty$?

7. Use the method of separation of variables to solve the following wave equation

$$u_{tt} = u_{xx}, 0 < x < \pi$$

$$u(x,0) = \cos(2x), u_t(x,0) = 1$$

$$u_x(0,t) = 0, u_x(\pi,t) = 0$$

8. Use the method of separation of variables to solve the following laplace equation

$$u_{xx} + u_{yy} = 0, 0 < x < 1, 0 < y < 1$$

$$u(x,0) = x, u(x,1) = 0, 0 < x < 1$$

$$u(0,y) = 0, u(1,y) = 0, 0 < y < 1$$