MATH256-201/202 Homework Assignment 3 (Due Date: Feb. 28, 2018)

Homework is admitted until 1pm on Feb. 28, 2018. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. For the following matrices, (a) Compute the eigenvalues; (b) Compute the associated eigenvectors

$$(1)\ \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix},\quad (2)\ \begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix},\quad (3)\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix},\quad (4)\ \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix}$$

2. Using the Abel's formula $W = Ce^{\int \text{trace}(A)dt}$ to compute the Wronskian for the following system of equations

$$(1)\ \mathbf{x}^{'} = \begin{pmatrix} e^t & e^t & \sin t \\ \sin t & 1 & \cos t \\ e^t & e^{-t} & -e^t \end{pmatrix}\mathbf{x},\quad (2)\ t\mathbf{x}^{'} = \begin{pmatrix} e^t & e^t & \sin t \\ \sin t & 1 & \cos t \\ e^t & e^{-t} & -e^t \end{pmatrix}\mathbf{x}$$

3. Solve the following initial value problem

$$(1)\ \mathbf{x}^{'} = \begin{pmatrix} 2 & -1 \\ -2 & 3 \end{pmatrix}\mathbf{x}, \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\quad (2)\ \mathbf{x}^{'} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}\mathbf{x}, \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ (3)\ \mathbf{x}^{'} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}\mathbf{x}, \mathbf{x}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

4. For the following system (a) find general solutions to the following systems, (b) classify the types (saddle, node (proper or improper), spiral, center) and stability (stable/unstable), (c) draw phase portrait of the trajectories

$$(1)\ \mathbf{x}^{'} = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix}\mathbf{x},\ (2)\ \mathbf{x}^{'} = \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix}\mathbf{x},\ (3)\ \mathbf{x}^{'} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}\mathbf{x},\ (4)\ \mathbf{x}^{'} = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}\mathbf{x},\ (5)\ \mathbf{x}^{'} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\mathbf{x}.$$

5. Solve the following system, assuming that $t > 0$

$$(1)\ t\mathbf{x}^{'} = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\mathbf{x},\quad (2)\ t\mathbf{x}^{'} = \begin{pmatrix} -3 & -2 \\ 1 & -1 \end{pmatrix}\mathbf{x}$$

Hint: Try $\mathbf{x} = \xi\ t^r$.

6. Consider the second order linear equation

$$ay^{''} + by^{'} + cy = 0$$

where a, b, c are constants. Let $x_1 = y, x_2 = y^{'}$. Write as $\mathbf{x}^{'} = A\mathbf{x}$. Show that the eigenvalues of A solve $ar^2 + br + c = 0$.

7. Consider the following system

$$\mathbf{x}^{'} = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}\mathbf{x}$$

Let $\rho = \sqrt{x_1^2 + x_2^2}, \theta = \arctan(\frac{x_2}{x_1})$. Compute $\rho^{'}$ and $\theta^{'}$ and find out the differential equations for ρ and θ. Draw phase portrait of the trajectories. Hint: $\rho\rho^{'} = x_1x_1^{'} + x_2x_2^{'}$, $\theta^{'} = \frac{-x_2x_1^{'} + x_1x_2^{'}}{x_1^2 + x_2^2}$.