MATH 256 Written Assignment 2

You may use Wolfram Alpha to evaluate any integrals.
Otherwise, you must show all your working.

1. Find the solution to each of the following ODEs:
   (a) $y'' + 3y' - 4y = 0$ with $y(0) = 1$ and $y'(0) = 0$.
   (b) $y'' + 2y' + 2y = 0$ with $y(0) = 1$ and $y'(0) = 0$.
   (c) $y'' - 4y' + 4y = 0$ with $y(0) = 1$ and $y'(0) = 0$.
2. For the following ODEs, find the value of the Wronksian when $x = 1$ given the value of the Wronsian at the value of x given. Do not solve the ODEs.
   (a) $y'' + xy' + y = 0$ given that the Wronskian is 1 when $x = 0$.
   (b) $x^2y'' + xy' + x^2y = 0$ given that the Wronskian is 2 when $x = 4$.
   (c) $xy'' + 2y' + e^x y = 0$ given that the Wronskian is 3 when $x = 2$.
   (d) $\sin(x)y'' - \cos(x)y' + \sin(x)y = 0$ given that the Wronskian is 1 when $x = \pi/2$.
3. For the ODE $x^2y'' + 4xy' - 4y = 0$, are the provided functions $y_1(x)$ and $y_2(x)$ a fundamental set of solutions for $x > 0$? Either prove that they are, or explain why they are not.
   (c) $y_1(x) = x$ and $y_2(x) = x^{-4}$.
   (d) $y_1(x) = x$ and $y_2(x) = 2x$.
   (e) $y_1(x) = x$ and $y_2(x) = x^{-1}$.
4. For the following ODEs, find a second solution $y_2(x)$ given the solution $y_1(x)$ provided.
   (a) $x^2y'' + 2xy' - 2y = 0$ given $y_1(x) = x$.
   (b) $(x - 1)y'' - xy' + y = 0$ given $y_1(x) = e^x$.
   (c) $xy'' - (x + 2)y' + 2y = 0$ given $y_1(x) = e^x$.
5. Find the general solution to each of the following ODEs. Hint: have a look at Q1.
   (a) $y'' + 3y' - 4y = 4x + 1$.
   (b) $y'' + 3y' - 4y = 10\cos(2x)$.
   (c) $y'' + 3y' - 4y = 5e^x$.
6. Find the general solution to each of the following ODEs. Hint: have a look at Q1.
   (a) $y'' + 2y' + 2y = e^x$.
   (b) $y'' + 2y' + 2y = x + 1 + e^x$.
   (c) $y'' + 2y' + 2y = e^{-x}\sin(x)$.
7. Find the general solution to each of the following ODEs. Hint: have a look at Q1.
   (a) $y'' - 4y' + 4y = xe^x$.
   (b) $y'' - 4y' + 4y = 4\cos(2x) + 5\sin(x)$.
   (c) $y'' - 4y' + 4y = e^{2x}$.