Chapter 7

Consider $X' = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix} X + \begin{pmatrix} 3t-2 \\ e^{-t} \end{pmatrix}$ —(1)

1. Use method of diagonalization to solve (1)
2. Use method of undetermined coefficients to solve (1)
3. Use method of variation of parameters to solve (1)
4. Find $\Phi(t)$ of $X' = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix} X$ such that

   $\Phi(0) = I$

5. Find the general solution of

   $x' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$.

6. Find the general solution of

   $x' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} + \begin{pmatrix} t^{-3} \\ -t^{-2} \end{pmatrix}$.

7. Solve $x' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} \csc t \\ \sec t \end{pmatrix}$, $\frac{\pi}{2} < t < \pi$

8. Compute Wronskian of $X' = \begin{pmatrix} t & e^{t^2} \\ \frac{1}{t^2+1} & \ln t \end{pmatrix} X$, $t > 0$.

Chapter 6

1. Use method of Laplace Transform to solve

(a) $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1$

(b) $y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0.$

(c) $y'' + 4y = \begin{cases} 1, & 0 < t < \pi \\ 0, & \pi < t < \infty \end{cases}, \quad y(0) = 1, \quad y'(0) = 0$

(d) $y'' + y = \begin{cases} t, & 0 \le t < 1 \\ 2 - t, & 1 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases}, \quad y(0) = 0, \quad y'(0) = 0$

2. Use method of Laplace Transform to solve

(a) $y'' + 3y' + 2y = \cos t, \quad y(0) = 0, \quad y'(0) = 1$

(b) $y'' + 2y' + 2y = 3u_1(t), \quad y(0) = 1, \quad y'(0) = 1$

(c) $y'' + 4y' + 5y = u_2(t) + (\sin t)\,\delta(t-3), \quad y(0) = 0, \quad y'(0) = 0$

(d) $y'' + 2y' + 5y = e^{t} + e^{2t}\delta(t-4) - 10u_5(t), \quad y(0) = 0, \quad y'(0) = 0$