Oct. 28

- HW5 / Quiz 3 Returned
- HW6 Solutions posted
  - return on Friday
- See Lab 7 for midterm practice.

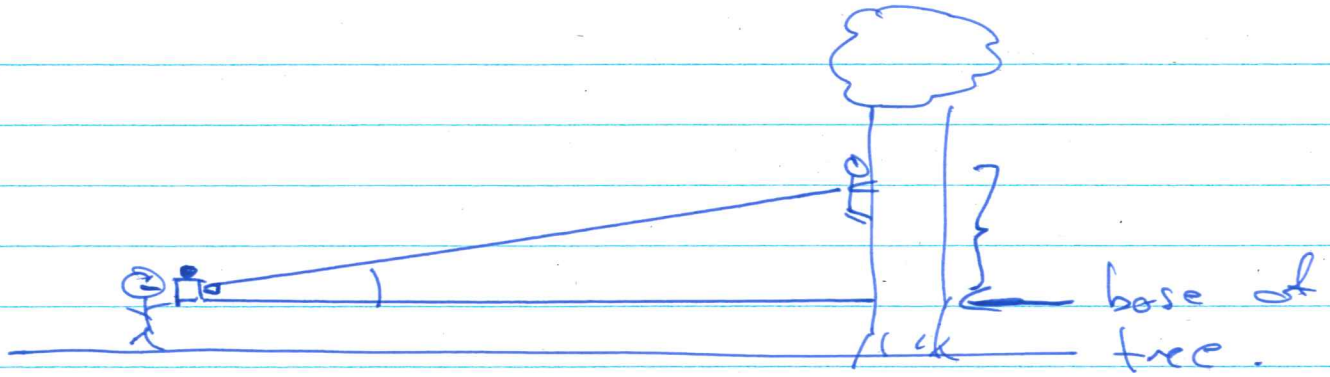
Office Hours:

Wed: 11-1pm in MATX 1102
Ian → 1-2pm in MATX 1118
Thurs: 3:30-5pm in MATX 1102
Fri: 1-3 pm in LSK 300C
Ian. → 3-4pm in LSK 300C.

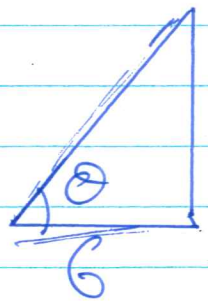
One last Related Rates Example.

You lumberjack friend is climbing a tree at a constant speed of 0.5 m/s.
You stand 6 meters from the base of the tree and film it with your camera.
How fast is the camera's angle changing when your friend is 8m above ~~your~~ from the base of the tree.

(2)



1. Picture / Notation.



SOHCAHTOA.

θ and L are functions of time.

$\theta = \theta(t)$
$L = L(t)$.

2. Given: $\frac{dL}{dt} = \frac{1}{2}$ m/s.

Want: $\frac{d\theta}{dt} = ?$

3. Equation: $\tan\theta = \frac{o}{a} = \frac{L}{6}$.

(3)

4. Chain Rule.

Take $\frac{d}{dt}$ of both sides.

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{L}{6}\right).$$

$$= \frac{1}{6}\frac{d}{dt}(L).$$

$$= \frac{1}{6}\frac{dL}{dt}.$$

What is $\frac{d}{dt}(\tan\theta)$. ?

What is $\frac{d}{d\theta}\tan\theta$?

$= \frac{d}{d\theta}\left(\frac{\sin\theta}{\cos\theta}\right) =$ ← Q.R.

$$\frac{\cos^2\theta \mp (-\sin\theta)\sin\theta}{\cos^2\theta}.$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}.$$

$$= \frac{1}{\cos^2\theta}.$$

(4)

some unknown function of t.

$$\frac{d}{dt}(\tan\theta) = \frac{1}{\cos^2\theta}\cdot\frac{d\theta}{dt}$$

chain rule.

$f(t) = \tan t$

$g(t) = \theta(t)$

$h(t) = \tan\theta = f(g(t))$

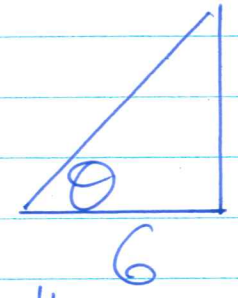
$f(t) = \tan t$

$g(t) = \theta(t)$.

So, $\frac{1}{\cos^2\theta}\frac{d\theta}{dt} = \frac{1}{6}\frac{dL}{dt}$ ← 0.5 cm/s

5. Solve/Substitute.

$$\frac{d\theta}{dt} = \frac{1}{6}\cos^2\theta\,\frac{dL}{dt}$$



$$= \frac{1}{6}\cos\theta\,(0.5)$$

want this angle when friend is 6 m above the base

$\Downarrow$

$\theta = \pi/4$.

(5)

$$\tan\theta = \frac{o}{a} = \frac{6}{6} = 1.$$

$$\tan\theta = 1. \qquad \text{unit circle.}$$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\sin\theta = \cos\theta.$$

$$\theta = \pi/4.$$

Sub ->

$$\frac{d\theta}{dt} = \frac{1}{6}\cos^2\left(\frac{\pi}{4}\right)\frac{1}{2}.$$

$$= \frac{1}{6}\left(\frac{1}{\sqrt{2}}\right)^2 \frac{1}{2}.$$

$$= \frac{1}{6}\cdot\frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{1}{24} \text{ radians/second}$$

$$\sim 2.4^\circ/s.$$

Review: Derivatives you need to know:

$f(x)$	$f'(x)$
x^n	nx^{n-1} (Power Rule).
$\sin x$	$\cos x$
$\cos x$	$-\sin x$.
e^x	e^x
$\ln x$	$1/x$.
$f(x)\cdot g(x)$	$f'(x)g(x) + f(x)g'(x)$. (product rule.)
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$. (Quotient Rule).
$f(g(x))$	$f'(g(x)) \cdot g'(x)$. (Chain rule)

OR Limit Def.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

(7)

Chain Rule.: Find:

$$\left(\sin\left(\underbrace{e^{2x}}_{\text{inside.}}\right)\right)'$$

$$= \cos\left(e^{2x}\right)\left(\underbrace{e^{2x}}_{\text{inside.}}\right)'$$

$$= \cos\left(e^{2x}\right) e^{2x} \cdot 2.$$

$$= 2\underbrace{e^{2x}}\cos\left(e^{2x}\right)$$

What happens at $x \to -\infty$.

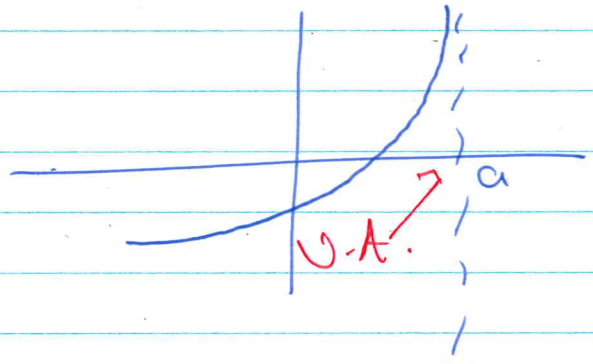
$y = 0$.

Asymptotes:

Q) What does it mean if $f(x)$ has a vertical asymptote at $x = a$.

A) either (or both)

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \pm\infty.$$



Example

To find V.A. ? $f(x) = \dfrac{x-3}{x+2}$.

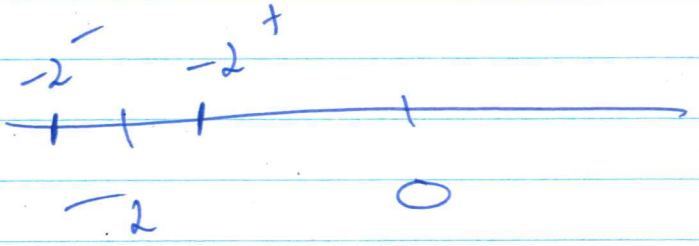
Identify candidates: $x = -2$?

- $\lim_{x \to -2^-} f(x) = +\infty$. $\left(\dfrac{-}{-}\right)$

- $\lim_{x \to -2^+} f(x) = -\infty$. $\left(\dfrac{-}{+}\right)$

$\Rightarrow$ V.A. at $x = -2$.

(9).



a) What does it mean for a func. to have H.A.

$\lim_{x \to \infty} f(x) = L$ ← a number.

or

$\lim_{x \to -\infty} f(x) = L$.