1

Oct. 14.

- HW 4 Due today
- HW 5 (short) Due Monday.
- Labs as usual this week.
- Quiz #2 marked.

(a) $\lim_{x \to 4} 2x^2 - 7x - 4$

$$\left(\cancel{2x^2 - 7x - 4 = 0 \quad \vdots} \right).$$

$$= 2(4)^2 - 7(4) - 4.$$

- Quiz #3 : Oct. 23
- Midterm : Nov. 2, (in class)

Recall the definition of the derivative (§ 2.1).

$$\boxed{\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

The derivative gives the slope of the tangent line (speed) to $f(x)$ at a given point x.

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Example: Find the derivative of $f(x) = x^3$ and use it to find the slope of the tangent line at $x = 2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}.$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}.$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}.$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2.$$

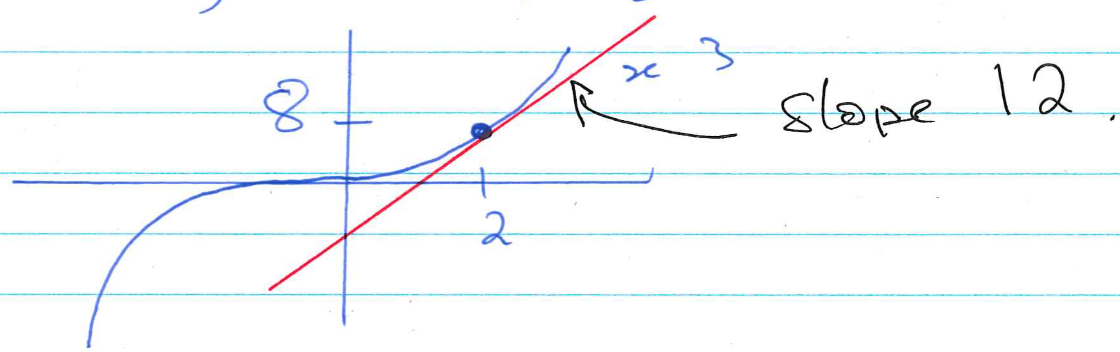
$$= 3x^2 + 0 + 0.$$

$$= 3x^2$$

So, the derivative of x^3 is $3x^2$.

The slope of the tangent line at $x = 2$ is:

$$f'(2) = 3(2)^2 = 3 \cdot 4 = 12.$$



Can we find the equation of the tangent line?

We have slope $m = 12$ and point $(2, 8)$.

$$y - y_1 = m(x - x_1)$$

$$y - 8 = 12(x - 2)$$

(or use $y = mx + b$.)

Let's collect what we have:

$$\cancel{(2x)}$$

$$(x^2)' = 2x$$

$$(x^3)' = 3x^2.$$

Any guesses for

$$(x^4)' = 4x^3.$$

In general we have the Power Rule.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

for any real number n.

The proof goes like this:

$$\frac{d}{dx}(x^n) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}.$$

a bunch of algebra.

$$= \cdots\cdots$$

$$= nx^{n-1}.$$

Example: Use power rule to find the derivative of
$f(x) = \sqrt{x}$.

(Note: this is done in the Lab using limits).

$$f(x) = \sqrt{x} = x^{1/2}.$$

$$f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}.$$

Example: Find the derivative of

$$f(x) = \frac{1}{x}.$$

(again, done in Lab with limits).

$$f(x) = \frac{1}{x} = x^{-1}.$$

$$f'(x) = -1\, x^{-2} = \frac{-1}{x^2}.$$

Clicker Q: What is the derivative of $f(x) = 5$?

→ A) 0

B) 5

C) $5x$

D) $1/5x$

Power Rule:

$f(x) = 5 = 5x^0$.

$f'(x) = 0 \cdot 5x^{-1}$

$= 0$.

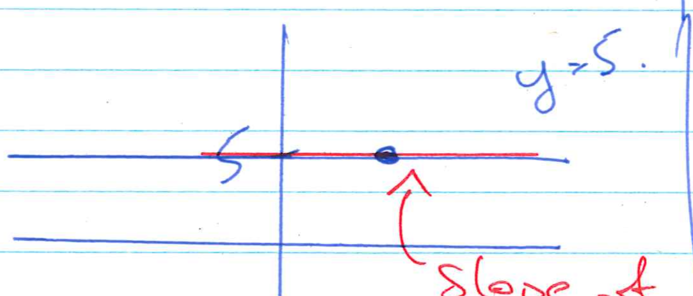
Limit Definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5 - 5}{h}$$

$$= \lim_{h \to 0} \frac{0}{h}.$$

$$= 0.$$



Slope of tangent is zero.

~~Clicker Q~~: Find $f'(x)$ for $f(x) = x^{\pi}$.

$f'(x) = \pi x^{\pi - 1}$.

Example: Find the derivative of

$$f(x) = \frac{1}{x^2} + x^{3/2} - x^{2015} + e^{\pi}$$

(when your function is the sum or difference of different functions you can just take the derivative of each)

$$f'(x) = -2x^{-3} + \frac{3}{2}x^{1/2} - 2015x^{2014} + 0.$$

Note: the derivative of any constant is zero.

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Clicker Q: Can we use power rule to take the derivative of

$$f(x) = 2^x \ ?$$

A) Yes
B) No
C) Don't know.

2^x is an exponential function not a polynomial.

$$f'(x) = 2^x \ln(2).$$

In general if

$$f(x) = a^x$$
$$f'(x) = a^x \ln(a).$$

In particular,

$$f(x) = e^x$$

this function is its own derivative.

$$f'(x) = e^x \ln(e^1)$$
$$= e^x.$$