① Nov. 9

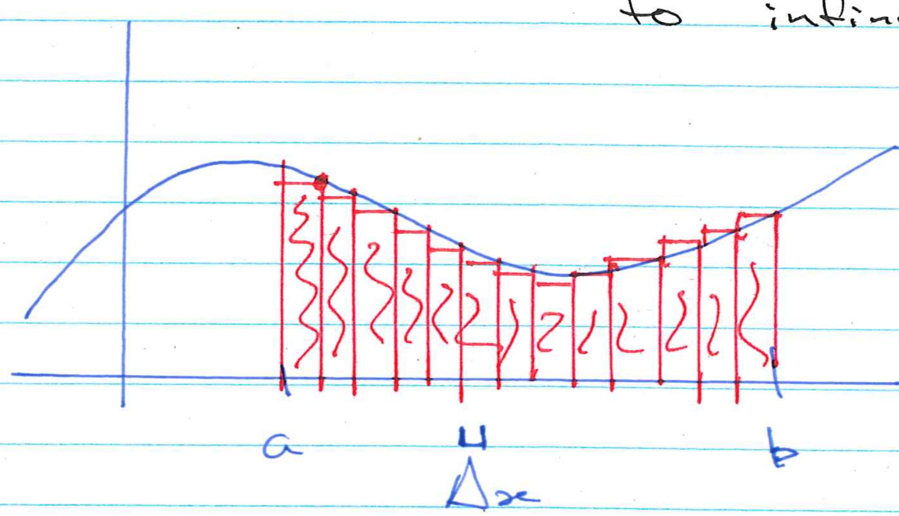
- Midterm Returned
  - grades posted.
  - Average: 62.5 %
  - Median: 60 %.
- Quiz Friday.
  - 1 of 3 Lab 8 problems.
- HW 8 Due Monday.

Definite Integrals:

Last class we defined the definite integral as:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x.$$

number of bars goes to infinity.



$f(x)$.

$$\Delta x = \frac{b-a}{n}.$$

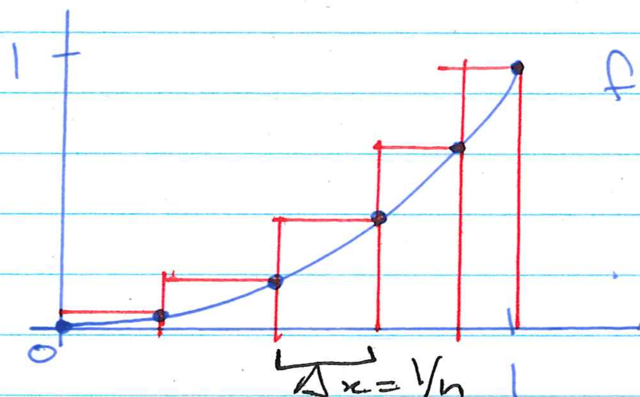
as $n \to \infty$

$\Delta x \to dx$

$\sum \to \int$.

Let us try to find the area under the curve $f(x) = x^2$. ~~for~~ from 0 to 1.

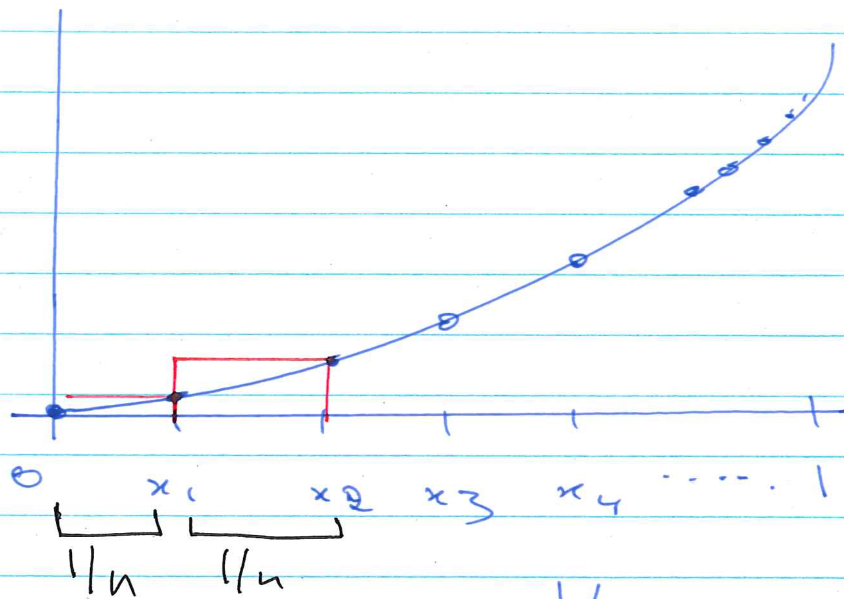
$$\int_0^1 x^2\,dx = \underbrace{\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x}_{\text{try to compute this limit.}}$$



Last week we approximated the area with a finite number of bars.
We will take the limit as $n \to \infty$.

Let's take n bars.

$$\Delta x = \frac{b-a}{n} = \frac{1}{n}.$$



$f(x) = x^2$,

$x_1 = 1/n$, $x_2 = 2/n$.

Clicker Q: What is the formula for x_i?

A) $1/i$ C) $1/n$

→ B) i/n D) Don't know.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x.$$

$$f(x_i) = f(i/n) = (i/n)^2.$$

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$$\Delta x = 1/n.$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\, \Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \frac{1}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^2}{n^3}.$$

$$= \lim_{n \to \infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2.$$

$$\left[\lim_{n \to \infty} \left(\frac{1^2}{n^3} + \frac{2^2}{n^3} + \frac{3^2}{n^3} + \ldots \right. \right.$$

$$\left. \left. \ldots \frac{n^2}{n^3}. \right) \right.$$

$$\left. \lim_{n \to \infty} \frac{1}{n^3} \left(1^2 + 2^2 + 3^2 + \ldots + n^2 \right) \right]$$

(5)

How to find:

$$\lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 .$$

Now, it turns out that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} .$$

We will not prove this but will check with $n=4$:

- $\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2$
  $= 1 + 4 + 9 + 16 = 30 .$

- $\frac{4(4+1)(2\cdot 4+1)}{6} = \frac{4\cdot 5\cdot 9}{6}$
  $= 2\cdot 5\cdot 3 = 30$

So, $\lim_{n\to\infty} \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) .$

$$\lim_{n\to\infty} \frac{n(2n^2+3n+1)}{6n^3}$$

$$= \lim_{n\to\infty} \frac{2n^3+3n^2+n}{6n^3}$$

$$= \lim_{n\to\infty} \frac{2+3/n+1/n^2}{6}$$

$\to 0$.

$$= 2/6 = 1/3$$ //

$$\int_0^1 x^2\,dx = 1/3.$$

Fundamental Theorem of Calculus:

There are a few parts to FTC. but basically it says that derivatives and integrals are inverse / oposite operations.

We will need the following result:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F'(x) = f(x)$.

We call $F(x)$ an anti-derivative.

So for our example:

$$\int_0^1 x^2\,dx \qquad f(x) = x^2. \quad F(x) = \frac{1}{3}x^3.$$

$$= F(1) - F(0)$$

$$= \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 = 1/3.$$

So $f(x) = x^2.$
$F'(x) = x^2.$

$$F(x) = \frac{1}{3}x^3.$$

Examples: $\int_1^3 \underbrace{(2x^2 + 3x - 1)}_{f(x).}\,dx.$

$F(x) =$

Want $F'(x) = f(x).$

(8).

$$F(x) = \frac{2}{3}x^3 + \frac{3}{2}x^2 - x$$

$$\left(\text{check:} \quad F'(x) = \frac{2\cdot\cancel{3}}{\cancel{3}}x^2 + \frac{3\cdot\cancel{2}}{\cancel{2}}x - 1 = 2x^2 + 3x - 1. \checkmark \right)$$

$$\int_1^3 f(x)\,dx = F(3) - F(1).$$

$$= \frac{2}{3}\cdot 3^3 + \frac{3}{2}3^2 - 3 - \left(\frac{2}{3}1^3 + \frac{3}{2}1^2 - 1\right).$$

//.

Also we could write

notation.

$$\left(\frac{2}{3}x^3 + \frac{3}{2}x^2 - x \Big|_1^3\right).$$

$$= \frac{2}{3}(3)^3 + \frac{3}{2}(3)^2 - 3 - \left(\frac{2}{3}(1)^3 + \frac{3}{2}(1)^2 - 1\right).$$