① Definite Integral

Nov. 13

Last class, we had,

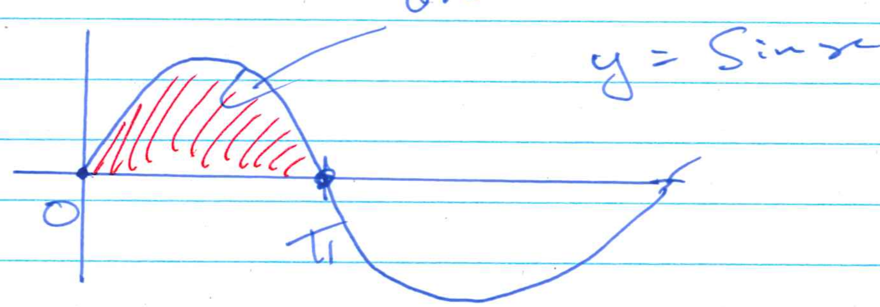
$$\int_a^b f(x)\,dx = F(b) - F(a).$$

where $F(x)$ is an anti-derivative of $f(x)$.

i.e. $F'(x) = f(x)$,

Examples:

1) $\int_0^{\pi} \sin x \, dx$ is 2.



$$f(x) = \sin x.$$
$$F(x) = -\cos x.$$

$$\left(\text{check}: F'(x) = (-\cos x)' = \sin x = f(x)\right)$$

①

$$\int_0^{\pi} \sin x \, dx = -\cos x \Big|_0^{\pi}$$

$$= -\cos(\pi) - \left(-\cos(0)\right).$$

$$= +1 + 1$$

$$= 2.$$

2) $\displaystyle\int_0^{2\pi} \cos x \, dx.$ $\qquad f(x) = \cos x$

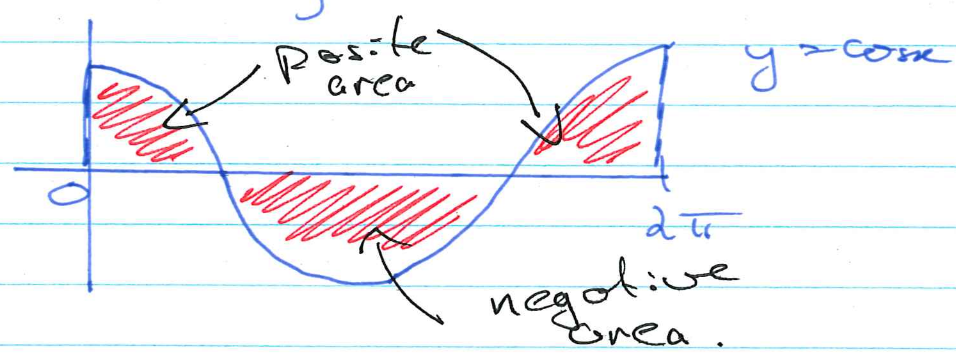
$\qquad F(x) = \sin x$

$$= \sin x \Big|_{x=0}^{2\pi}$$

$$= \sin(2\pi) - \sin(0)$$

$$= 0 - 0 = 0 /\!/.$$

Let's try and draw a picture:



3

If we wanted all the area to count as positive area we could find:

$$\int_0^{2\pi} |\cos x| \, dx.$$

3) $$\int_0^1 \left(e^{x} + e^{-x}\right) dx.$$

integral rule

$$= \int_0^1 e^{x}\,dx + \int_0^1 e^{-x}\,dx.$$

$-1/e + 1.$

$f(x) = e^{x}$

$F(x) = e^{x}$

$$\int_0^1 e^{x}\,dx = e^{x}\Big|_{x=0}^{1}$$

$$= e^{1} - e^{0}$$

$$= e - 1.$$

(4).

$$\int_0^1 e^{-x}\,dx \qquad f(x) = e^{-x}$$
$$F(x) = -e^{-x}.$$

try $\quad F(x) = -e^{-x}.$
$\quad\;\;\; F'(x) = +e^{-x}.$

$$\int_0^1 e^{-x}\,dx = -e^{-x}\Big|_{x=0}^{1}$$
$$= -e^{-1} - (-e^{0}).$$
$$= -1/e + 1.$$

$$\int_0^1 \left(e^{x} + e^{-x}\right)dx = e - 1 - 1/e + 1$$
$$= e - 1/e.$$
$$\boxed{= e - e^{-1}}.$$

4). $\displaystyle\int_1^2 \frac{1}{x}\,dx.$ $\qquad f(x) = 1/x$
$F(x) = \ln x.$

$$= \ln(2) - \overbrace{\ln(1)}^{0} = \ln(2).$$