... Continuing can example of last day...

$$dV = 2\pi \cdot 2 \cdot 10 \cdot (0.05) + \pi r^2 (0.1 + 0.1)$$

Note $\longrightarrow$ $= 40\pi\,(0.05) + 4\pi\,(0.2)$

$$= 2\pi + 0.8\pi = 2.8\pi = 8.8\,\text{cm}^3$$

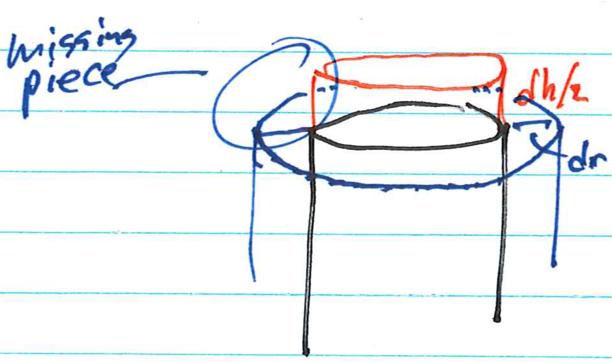
Sensitivity analysis: $40\pi \gg 4\pi \Rightarrow$ amount of metal used to make the can is <u>more</u> <u>sensitive</u> to change in the radius than to change in height.

<u>Exact</u> <u>Sol'n</u>: ~~V~~ $V(r,h) = \pi r^2 h$

$$V(r+dr,\, h+dh) - V(r,h) = \ldots =$$

$$0 + \underbrace{2\pi r h\,dr + \pi r^2 dh}_{dV} + \underbrace{2\pi r\,dr\,dh + \pi (dr)^2 (h+dh)}_{\text{small corrections}}$$

(recall the ε_1 and ε_2 in the <u>theory</u>)



Note: Recall example $V = xyz$, volume of a box.

$$dV = \left|\frac{\partial V}{\partial x} dx\right| + \left|\frac{\partial V}{\partial y} dy\right| + \left|\frac{\partial V}{\partial z} dz\right|$$

Max error? Sometimes need to add $|\cdot|$ abs values to each term

Note 2: we must do the modelling to interpret these results (e.g. factors of 2 in previous examples)

Midterm up to here.

review: matching eqns to surfaces/contour plots

Colins tricks for these kind of problems

① Symmetry: - radial symmetry: $f(x,y) = g(x^2+y^2)$ (r^2)

Contours are circles

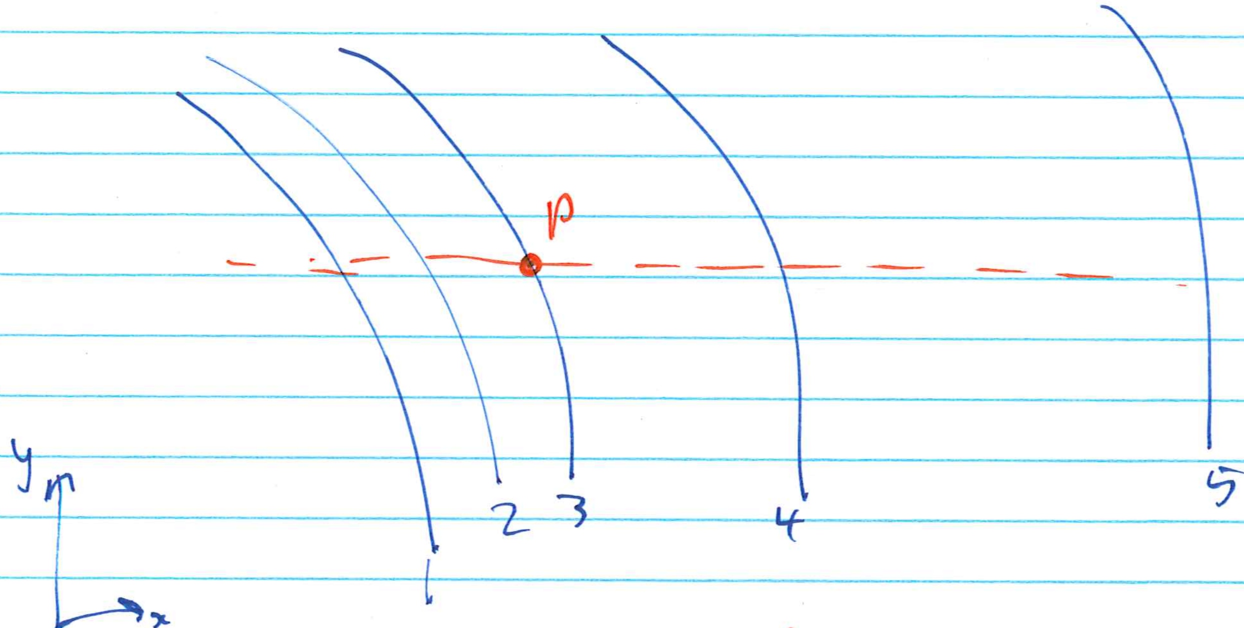
- dependence on only one variable $f(x,y) = g(y)$

② slice and sketch trace curves.

$x=0$, $y=0$, $z=k$

③ trig fcns / oscillations / periodic behaviour.

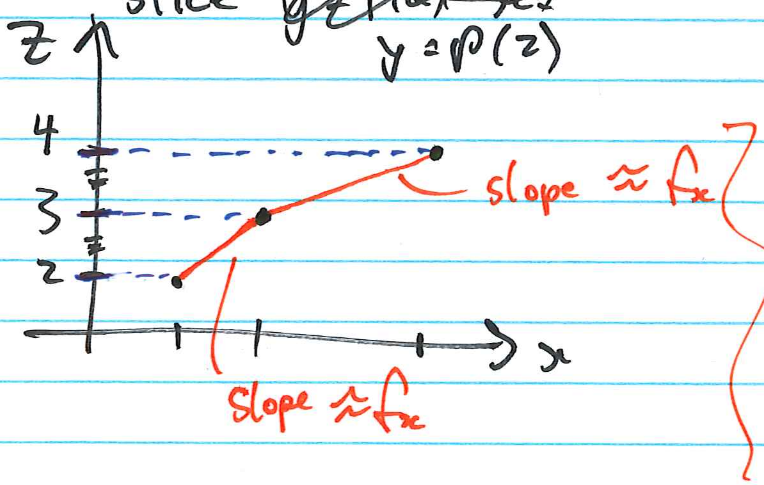
Can infer 2nd-deriv info from contour spacing.



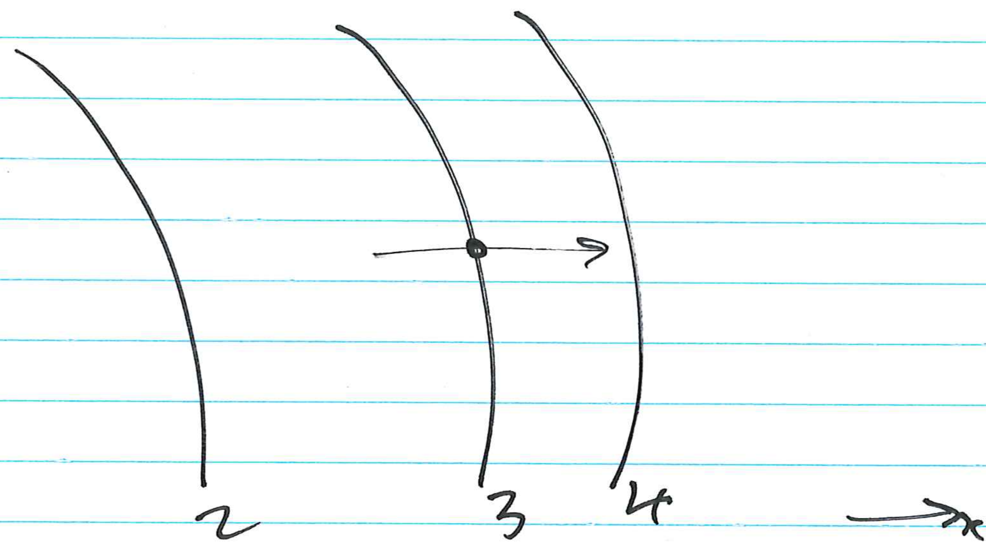
b/c $4 > 3$: $f_x > 0$

f_{xx} : are the contours becoming closer or further apart as $x \nearrow$.

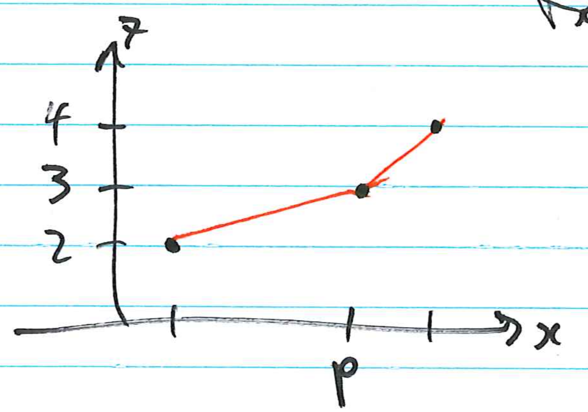
slice ~~$y = p(x)$~~ $y = p(2)$



$\Longrightarrow$ $f_{xx} < 0$ concave down.



$f_x > 0$



$f_{xx} > 0$ concave up

+ 2 more cases for $f_x < 0$ concave up
and $f_x < 0$ concave down.

Repeat same idea in y.