Last day: guitar strings and PDEs

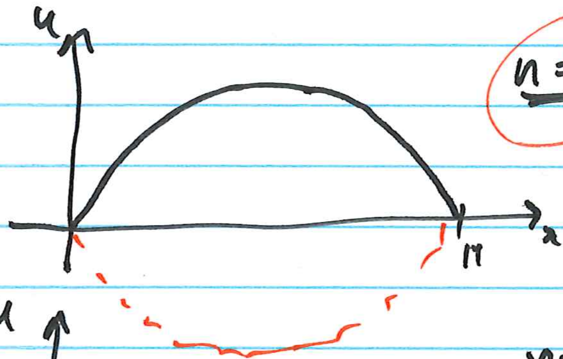
Wave Eqn: $u_{tt} = a^2 u_{xx}$ $a = \text{const} = \frac{T}{\rho}$

Exercise: show $u(x,t) = \cos(ant)\sin(nx)$ is a sol'n for any integer n

compute partials, sub in to the PDE

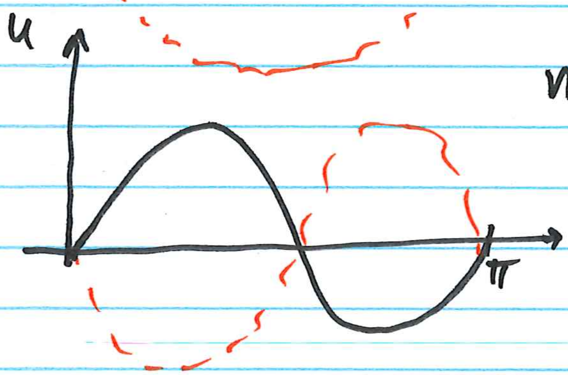
vibration in time

pattern



$n=1$

note



$n=2$

harmonics / overtones etc.

and initial shape of the wave

$\Longrightarrow$ interesting sounds

Ex Listen to "Prophets of Rage" Tom Morello, Chuck D.

"No Hatred f*ck racists"

Nonlinear PDE: Fisher-KPP eqn

$$\underbrace{u_t = u_{xx}}_{\text{diffusion}} + \underbrace{u(1-u)}_{\text{nonlinear}}$$

Reaction - diffusion

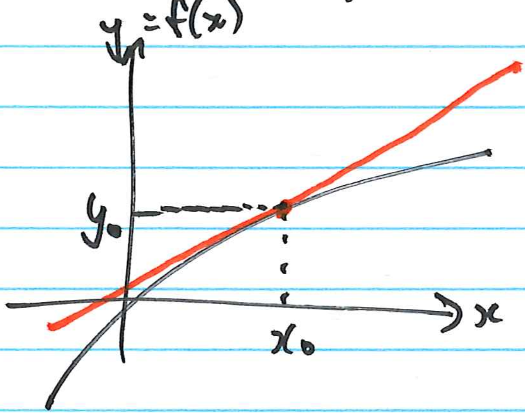
$$u_t = u_{xx} + u_{yy} + uv^2$$

$$v_t = \frac{1}{3}(u_{xx} + u_{yy}) - uv^2$$

Tangent Planes & Linear Approx.

Covered later in APEX book. §12.4?

Review single-variable calculus:



— tangent line at $x = x_0$

$\hookrightarrow$ $y = y_0 + f'(x_0)(x - x_0)$

tangent to the curve $y = f(x)$

Linear approx: $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

useful if x close to x_0

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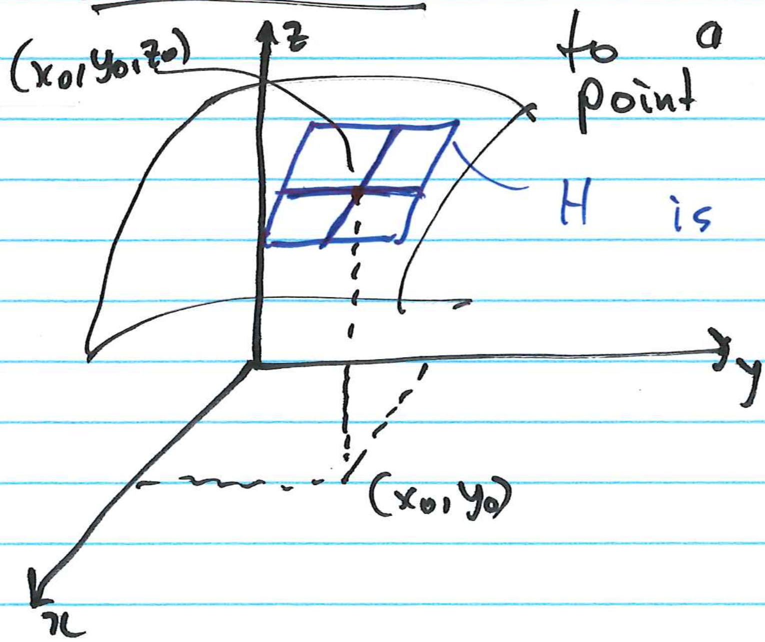
2 variable: want the best linear approx to a surface $z = f(x,y)$ at a point (x_0, y_0, z_0)



H is the tangent plane at (x_0, y_0, z_0): H contains the lines tangent to the two trace curves from $x = x_0$, $y = y_0$

Two ~~par~~ tangent lines:

$$z = z_0 + f_y(x_0, y_0)(y - y_0) \qquad (1)$$

$$z = z_0 + f_x(x_0, y_0)(x - x_0) \qquad (2)$$

It must contain these two lines

↳ $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

(For now) the tangent plane is not vertical

$$\Rightarrow \quad C \neq 0$$

$$z = z_0 - \frac{A}{C}(x - x_0) - \frac{B}{C}(y - y_0)$$

Slice $y = y_0$ and match with (2)

" $x = x_0$ " " " (1)

$$-\frac{A}{C} = f_x(x_0, y_0), \quad -\frac{B}{C} = f_y(x_0, y_0)$$

$\Rightarrow$ We have an equation for the tangent plane

$$\boxed{z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

Notes: linear

is tangent to $z = f(x, y)$ at (x_0, y_0, z_0)

Ex $z = f(x,y) = e^{-(x^2+y^2)}$

Find tangent plane at $(x_0, y_0) = (0,0)$

$$\frac{\partial f}{\partial x} = e^{-(x^2+y^2)}(-2x) = 0$$

$$\frac{\partial f}{\partial y} = -2y\, e^{-(x^2+y^2)} = 0$$

@ $(0,0)$

z $f(0,0) = 1$

Tangent plane: $z = \overset{1}{\cancel{z_0}} + \overset{0}{\cancel{f_x(0,0)}}(x-0) + \overset{0}{\cancel{f_y(0,0)}}(y-0)$

$$\boxed{z = 1}$$

Ex Same at $(x_0, y_0) = (1,1)$

$f(1,1) = e^{-2}$ $\quad f_x(1,1) = -2e^{-2}$

$f_y(1,1) = -2e^{-2}$

Tangent plane: $z = e^{-2} - 2e^{-2}(x-1) - 2e^{-2}(y-1)$

$$= e^{-2}\left[1 - 2x + 2 - 2y + 2\right]$$

$$= e^{-2}\left[5 - 2x - 2y\right]$$