2^{nd}- partial derivatives

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

Concavity of the trace curve in either x or y slices

please review this from last year!

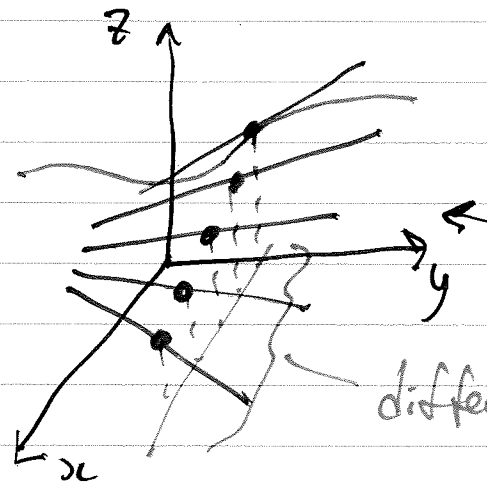
Now ~~the~~ are mixed partials:

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$

Meaning: f_{xy} : f_x is the slope of trace curve in a $y = \text{const}$ slice

f_{xy} is the rate of change of that slope f_x w.r.t. y.



Similarly for f_{yx}

different $x = \text{const}$ slices

Ex $f(x,y) = y^4 - xy^2 + x^3$

$f_x = 0 - y^2 + 3x^2 \qquad f_y = 4y^3 - 2xy + 0$

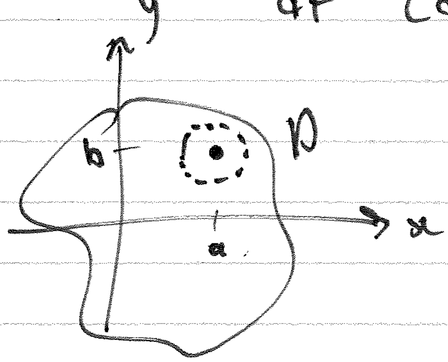
$f_{xx} = 0 + 6x \qquad f_{yy} = 12y^2 - 2x$

$f_{xy} = -2y + 0 \qquad f_{yx} = 0 - 2y$

Coincidence or conspiracy??

Fubini's ~~Conspiracy~~ Theorem

Suppose the domain D of f contains ~~(a,b)~~ a disc centred at (a,b). ~~If~~ f_{xy} and f_{yx} are each continuous on that disc, then



$$f_{xy}(a,b) = f_{yx}(a,b)$$

<u>Partial deriv example</u> "hold all other <u>indep vars</u> const, take deriv"

Suppose $f(x,y) = \int_x^y g(t)\,dt$

dummy variable of integration

F. T. C. $\quad = G(y) - G(x)$ where $G'(t) = g(t)$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(G(y)^{\nearrow 0} - G(x) \right)$$

$$= -G'(x)$$ careful with "prime" in multivariable calc.

$$= -g(x)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(G(y) - G(x)^{\nearrow 0} \right)$$

$$= G'(y) = g(y)$$

$\Downarrow$

$\cancel{f'(x,y)}$

ϕ

no meaning

PDEs: Partial differential equations

- not in the APEX book.
- notes on website.

<u>Ex</u> $u_{xx} + u_{yy} = 0$: find $u(x,y)$ that satisfies <u>Laplace's Eqn.</u>

eg: $u = 4x + 7y$, $u = xy$ } $u_{xx} = 0 = u_{yy}$

Solving PDEs is hard!!

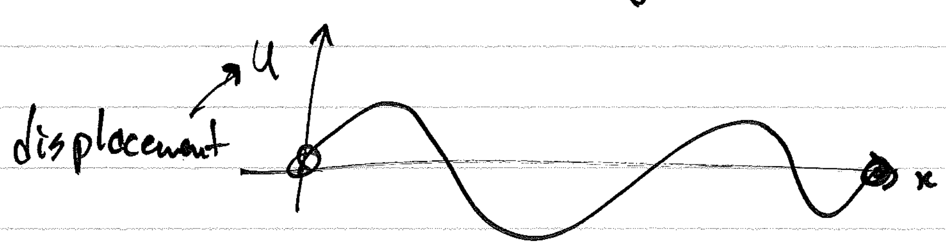
<u>But</u> <u>easy</u> to check if a given u solves the problem.

<u>Ex</u> Wave Eqn

$a = \frac{\text{tension}}{\text{density}}$

$$u_{tt} = a^2 u_{xx}$$, sol'n $u(x,t)$

model for guitar string



$$u(x,t) = \cos(ant) \sin nx$$