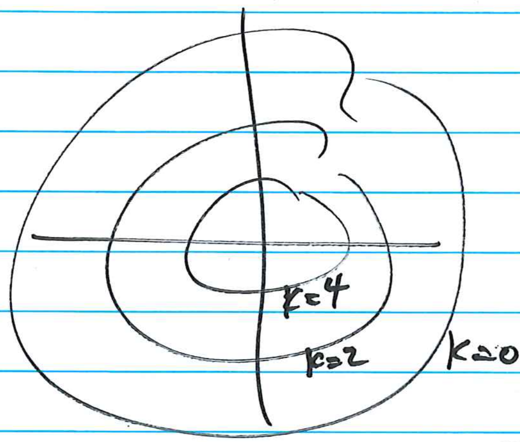
Contour plots ← usually the contours are equispaced

↓

sliced uniformly in z

(plot $f(x,y)=k$)



§12.3 Partial Derivatives

For a fcn of two independent variables $f(x,y)$, we can compute

dependent variable

a partial derivative by:

partial derir w.r.t. x

1. "fix" one of the variables, say y.
2. now we have a fcn of one var (x)
3. take "the" derivative w.r.t. x
4. "unfix" the other variable (y)

↳ result depends on x and y

find the partial derivative of f w.r.t. x and w.r.t. y.

Example: $f(x,y) = x^4 + 2x^2y^2 + 17 + y + e^{xy}$

fix y: $\frac{\partial f}{\partial x} = 4x^3 + 4xy^2 + 0 + 0 + e^{xy} \cdot y$

"e^{17x}"

fix x: $\frac{\partial f}{\partial y} = 0 + 4x^2y + 0 + 1 + e^{xy} \cdot x$

Notation $z = f(x,y)$

↳ partial derivs: (1) $\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} = f_x$

(2) $\frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} = f_y$

still fcns of (x,y)

Notice there are two derivatives of $f(x,y)$: f_x and f_y.

Ex using f above, what is $f_x(1,2)$ and $f_y(1,2)$?

$f_x(1,2) = 4 + 16 + 2e^2 = 20 + 2e^2$

$f_y(1,2) = 9 + e^2$

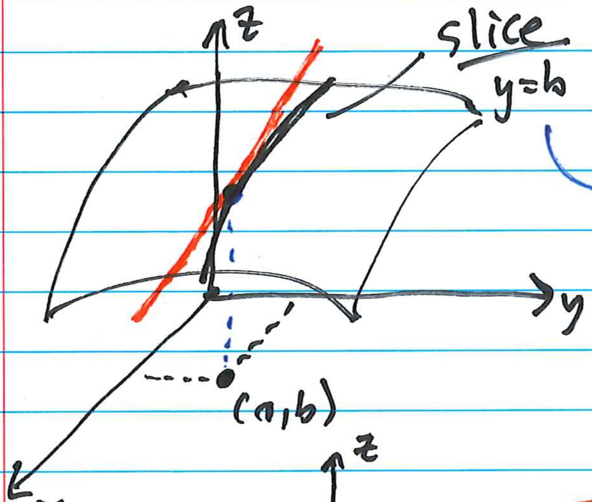
f_x and f_y are defined in terms of limits

$$f_x(x,y) = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

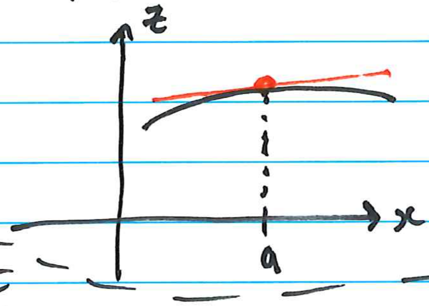
Geometric view of ~~part~~ partial derivs

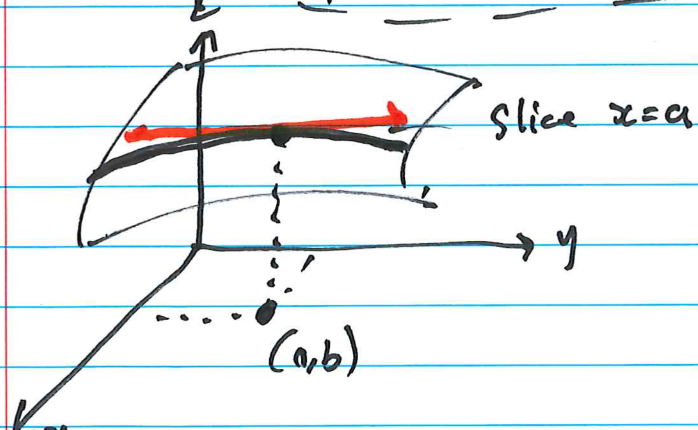
$\boxed{z = f(x,y)}$



Gives the trace curve $f(x,b)$

$f_x(a,b)$ slope of the tangent to the $y=b$ slice of $z=f(x,y)$ surface





$f_y(a,b)$ slope of the tangent to the $x=a$ slice of $z=f(x,y)$ surface

Ex $\quad z = \sqrt{1-x^2-y^2}$

$$\frac{\partial z}{\partial x} = \frac{1}{\cancel{2}}\left(1-x^2-y^2\right)^{-1/2}\left(-\cancel{2}x+0\right) = \frac{-x}{z}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}\left(\qquad\right)^{-1/2}\left(-2y\right) = \frac{-y}{z}$$

alt $\quad z^2 = 1-x^2-y^2, \Rightarrow x^2+y^2+z^2=1$

Implicit differentiation

Careful: $\frac{\partial}{\partial x}\left[x^2+y^2+z^2 = 1\right]$

$$2x + 0 + 0 = 0 \quad \times$$

no! remember: $\underbrace{z}_{\text{dep.}}(\underbrace{x, y}_{\text{indep.}})$

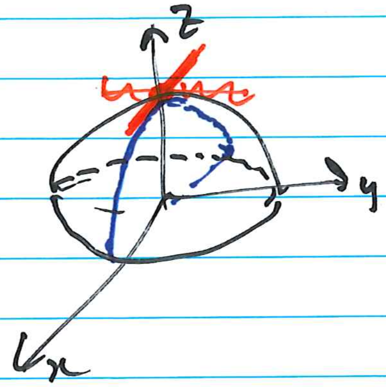
$$\frac{\partial}{\partial x}\left[x^2+y^2+z^2\right] = \frac{\partial}{\partial x}\left[1\right]$$

$$2x + 0 + 2z\frac{\partial z}{\partial x} = 0$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{x}{z}$$

Ex same $z = \sqrt{1-x^2-y^2}$, look at geometry

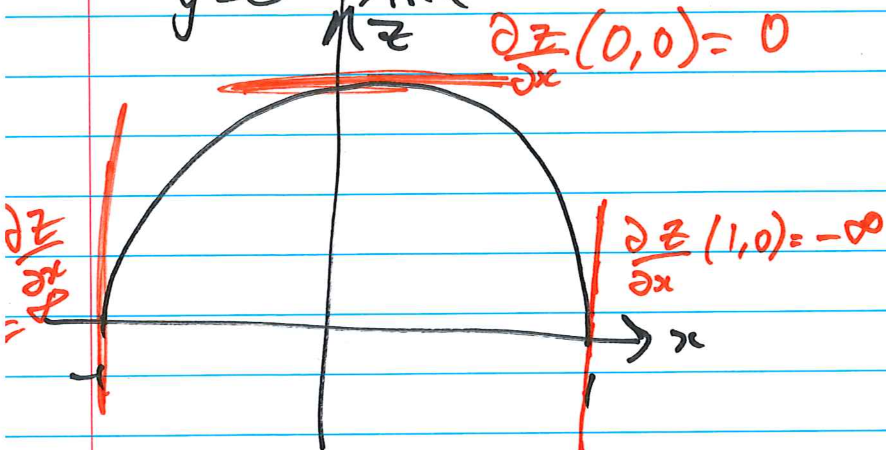
last week it was a hemisphere



$$\frac{\partial z}{\partial x}(x,y) = \frac{-x}{z}$$

$$\frac{\partial z}{\partial x}(0,0) = \frac{-0}{1} = 0$$

y=0 slice



$$\frac{\partial z}{\partial x}(1,0) = \text{"}\frac{-1}{0}\text{"} = -\infty$$

$$\frac{\partial z}{\partial x}(-1,0) = \text{"}\frac{1}{0}\text{"} = \infty$$

↳ can also look at 2^{nd} partial derivatives. We can do that in slices too (mostly!).

e.g. here $\frac{\partial^2 z}{\partial x^2} < 0$

↓

slice ~~that~~ y=0 is concave down.