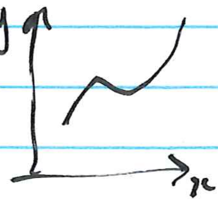
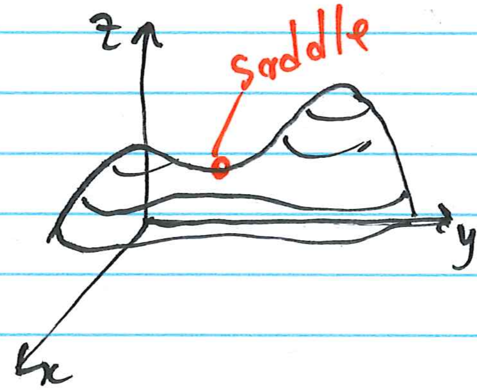
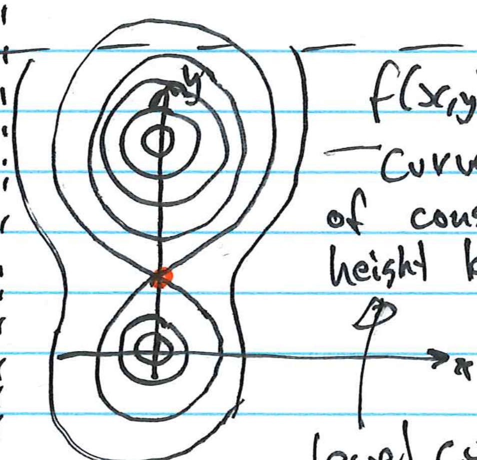
Review Plotting

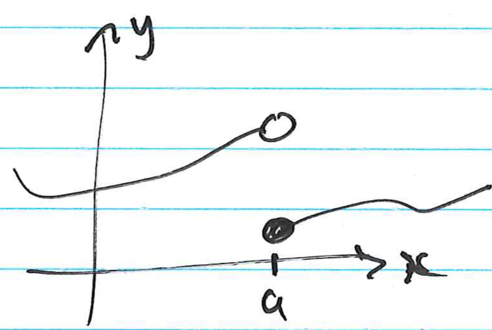
	$f(x)$	$f(x,y)$	$f(x,y,z)$
graph	new var y $y = f(x)$ 	new var $z = f(x,y)$ 	new var $w = f(x,y,z)$ 4D ← how to graph? ? x, y, z: 3D space w: density, temperature, concentration
contouring	? pts on a line with $f(x) = k$	$f(x,y) = k$ — curves of constant height k level curves	level surfaces $f(x,y,z) = k$

§ 12.2 Limits and Continuity

Calculus needs limits! Technically we need ε-δ proofs (oh no!), but intuitive understanding is important! (yes!)

Single variable graph $y = f(x)$

Ex1

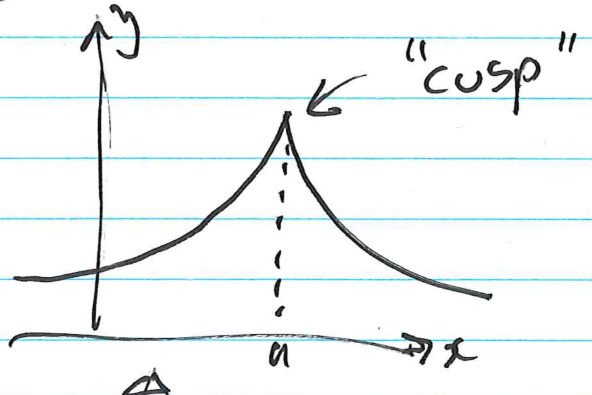


f is not continuous ("cts") at $x = a$ b/c

left-hand $\lim_{x\to a^-} f(x) \neq$ right-hand $\lim_{x\to a^+} f(x)$

So we say $\lim_{x\to a} f(x)$ does not exist (D.N.E.)

Ex2



f is cts

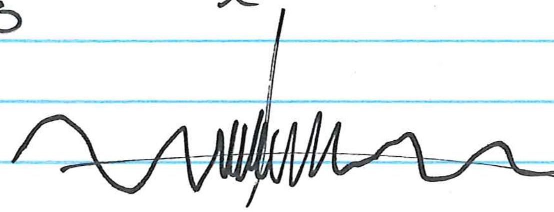
but $f'(a)$ DNE

(again left-hand lim $\neq$ right-hand lim)

"Lipschitz cts" ← roughly bounded left/right limits of derivative.

Ex 3 $\lim_{x \to \infty} \sin x$ DNE

Similarly $\lim_{x \to 0} \sin \frac{1}{x}$ DNE



Multivariable: $f(x,y)$

$$\lim_{(x,y)\to(a,b)} f(x,y)$$

New idea: many paths to (a,b), all must give the same limit.

Example $f(x,y) = \frac{xy}{x^2+y^2}$

What is the limit $\lim_{(x,y)\to(0,0)} f(x,y)$?

let's start with straight lines $y = mx$, $m \in \mathbb{R}$. $\Rightarrow$ 1D limit problem

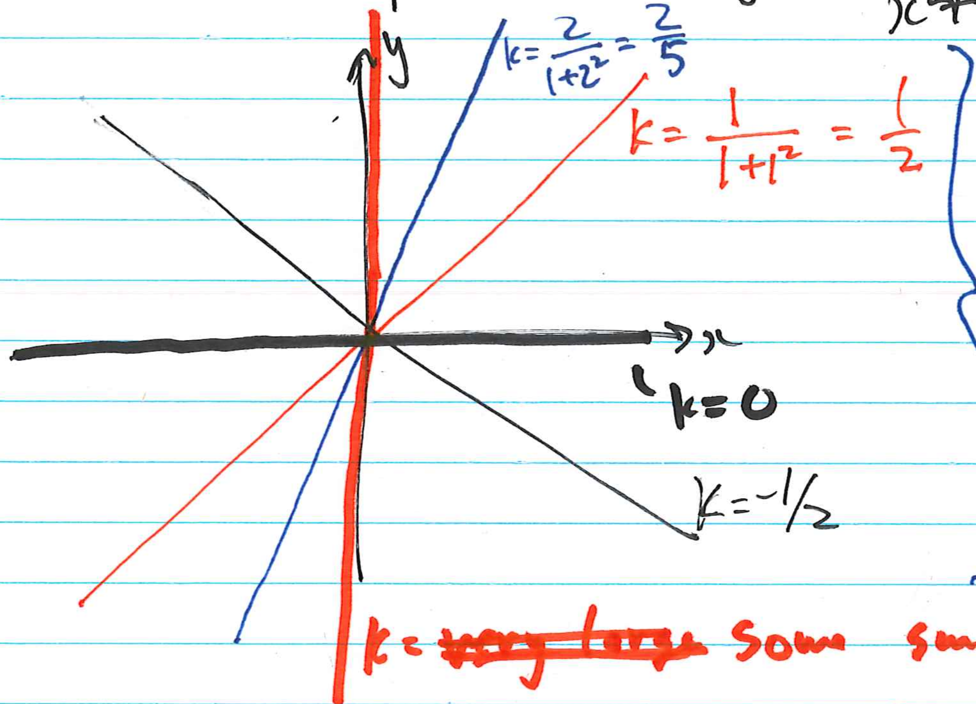
$$\lim_{x\to 0} f(x,mx) = \lim_{x \to 0} \frac{mx^2}{x^2+m^2x^2} = \lim_{x\to 0} \frac{m\cancel{x^2}}{(1+m^2)\cancel{x^2}}$$

$$= \frac{m}{1+m^2}$$

We have differenent values for the limit along different lines

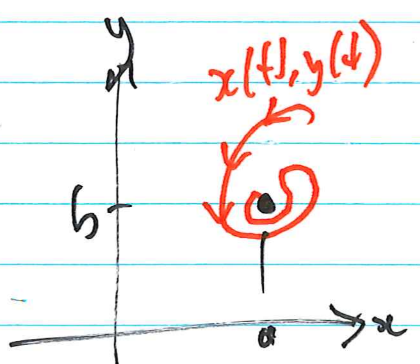
$\Rightarrow$ limit D.N.E.

Ex Contour plot $f(x,y) = \dfrac{xy}{x^2+y^2}$



Careful, getting this contour plot is nontrivial. You have to study the previous lim calculation very carefully!

Def'n The limit $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ exists and is equal to L if, for all continuous paths $x = x(t)$, $y = y(t)$ with $x(\tau) = a$ and $y(\tau) = b$, we have

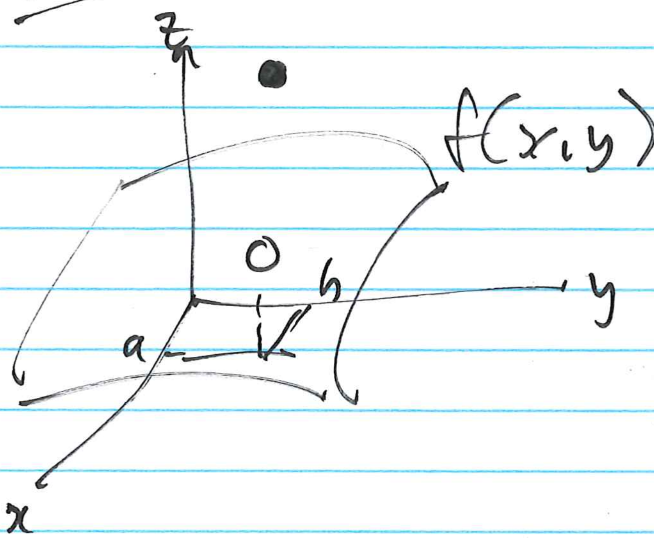


$$\lim_{t\to\tau} f(x(t), y(t)) = L \quad \text{(and exists)}$$

Def'n: $f(x,y)$ is continuous at (a,b) if

$$\lim_{(x,y)\to(a,b)} f(x,y) = L = f(a,b)$$

Ex



Here limit exists but $L \neq f(a,b)$

Note: checking path $y = mx$ can be used to show $f(x,y)$ not cts.

But harder to show $f(x,y)$ is cts!!

ε-δ (not examined)