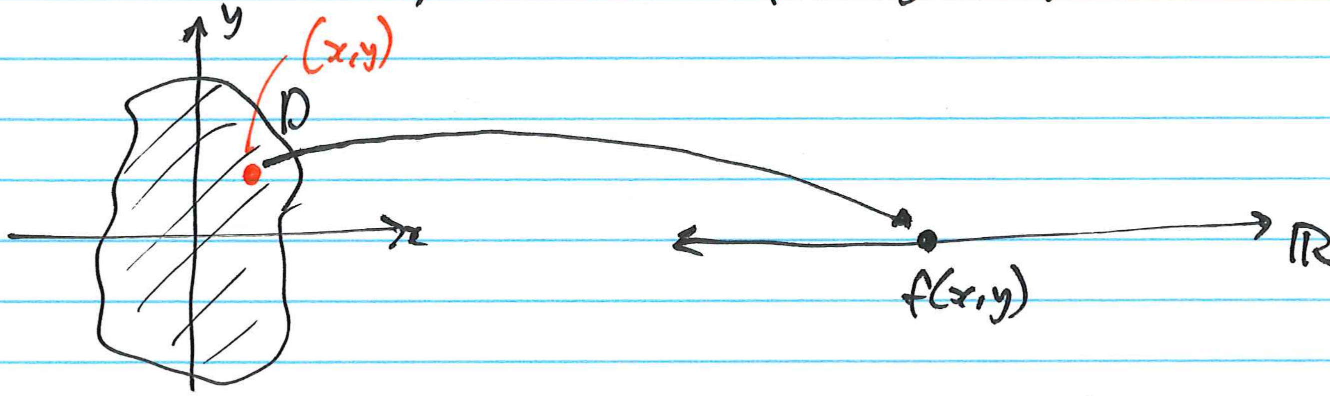
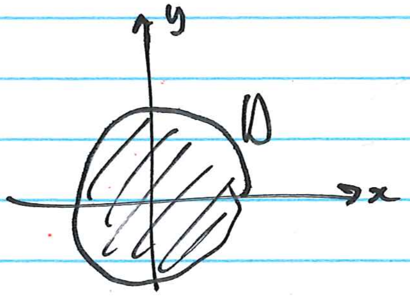
§12.1 Functions of several variables

$\hookrightarrow 2, 3$

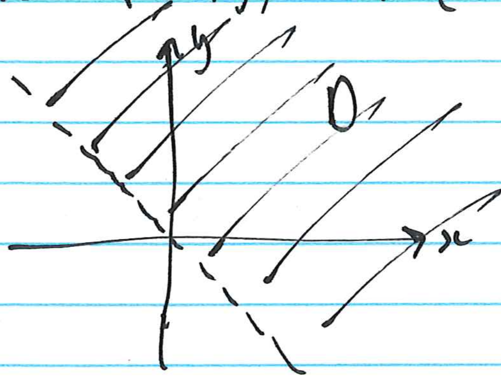
Def'n A function f of 2 variables with a domain $D \subseteq \mathbb{R}^2$ is a rule that assigns, to each point $(x,y) \in D$, a unique number $f(x,y) \in \mathbb{R}$



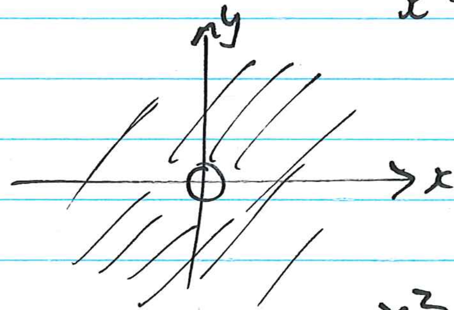
Ex (a) $f(x,y) = \sqrt{1-x^2-y^2}$, $D = \{(x,y) : x^2+y^2 \leq 1\}$



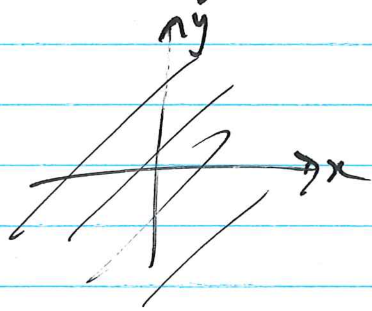
(b) $f(x,y) = \ln(x+y)$, $D = \{(x,y) : x+y > 0\}$



(c) $h(x,y) = \dfrac{1}{x^2+y^2}$, $D = \{(x,y) : (x,y) \neq (0,0)\}$

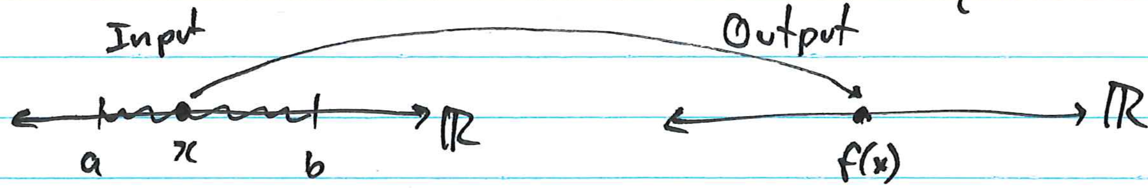


(d) $k(x,y) = e^{x^2+y} + \sin(y^4+3x)$, $D = \mathbb{R}^2$

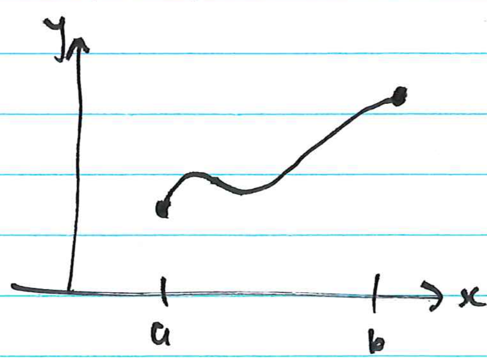


Graphs of fcns

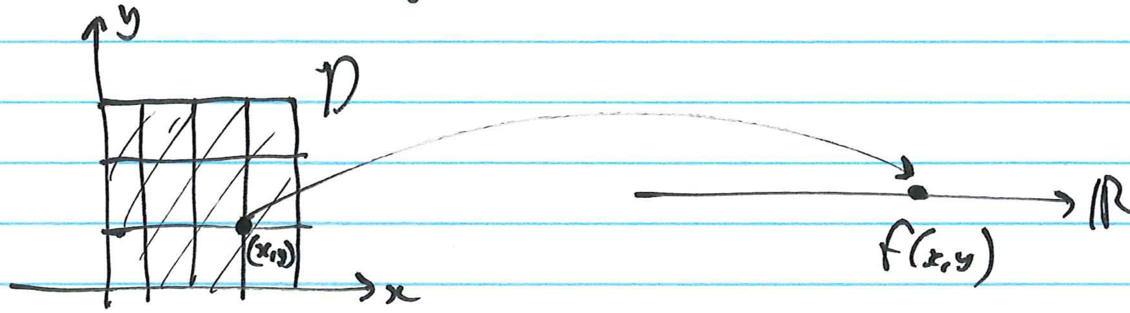
one-variable case : $D = \{x : a \leq x \leq b\}$

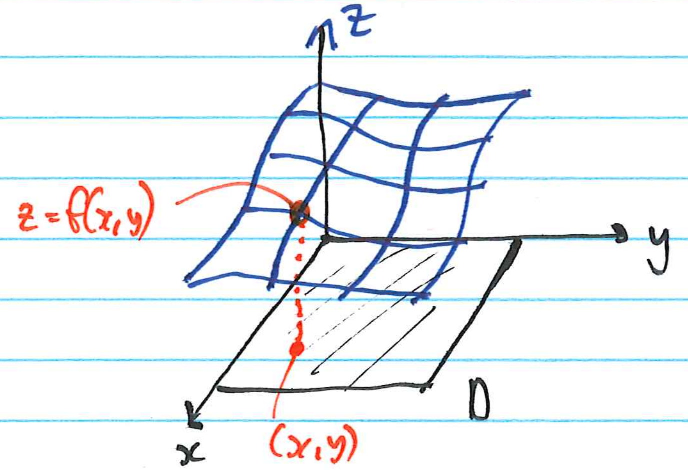


Graph $y = f(x)$



two-variable graph: introduce $z = f(x,y)$





graph is a surface with eqn:

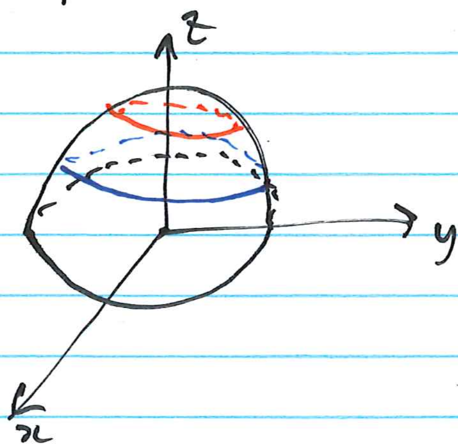
$$z = f(x,y)$$

<u>Example</u> $f(x,y) = \sqrt{1-x^2-y^2}$ $\qquad D = \{(x,y) : x^2+y^2 \le 1\}$

Graph: $z = f(x,y) = \underbrace{\sqrt{1-x^2-y^2}}_{\text{pos. root}} \implies z^2 = 1-x^2-y^2$

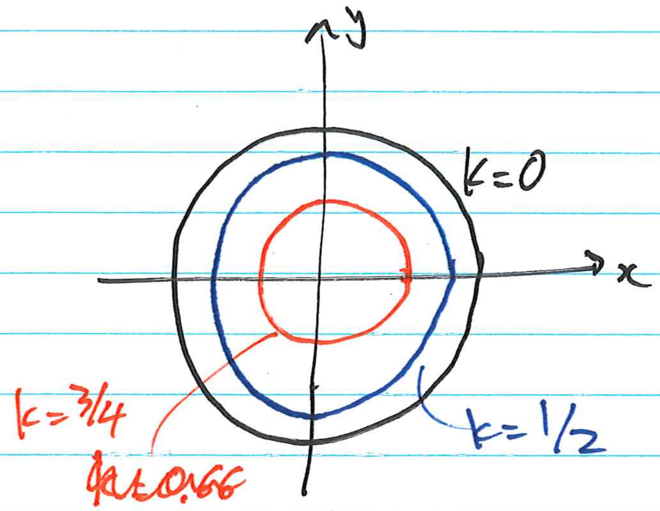
$x^2+y^2+z^2 = 1$

unit sphere



Contour plots : contours are curves in the x-y plane found by $f(x,y) = k$ for some values of k.

Example from above:



$k=0$: $f(x,y) = 0 = \sqrt{1-x^2-y^2}$

$\Rightarrow x^2+y^2 = 1$

$k=1/2$: $1/2 = \sqrt{1-x^2-y^2}$

$1/4 = 1-x^2-y^2$

$x^2+y^2 = \left(\sqrt{\tfrac{3}{4}}\right)^2 = r^2$

$(0.866)^2$

$k=3/4$

$(3/4)^2 = 1-x^2-y^2$

$x^2+y^2 = \left(\sqrt{7/16}\right)^2$

$(0.66)^2$

Summarize : - each k value is a curve corresponding to a slice of the 3D graph ~~at~~ through $\boxed{z=k.}$

- curves of constant height

↳ e.g. topographic map.

3- variables : $f(x, y, z)$

~~Contours?~~ isosurfaces / level surfaces:

$$\underbrace{f(x,y,z) = k}_{\text{diff. surfaces for each } k \text{ value}}$$

Eg $f(x,y,z) = \underbrace{x^2+y^2+z^2 = k}_{}$

isosurfaces are spheres of radius $\sqrt{k}$