Last day: dot product & projection

- vector projection of $\vec{b}$ onto $\vec{a}$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$$

- scalar projection $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

Note: example last day didn't work
↳ need a vector $\perp$ to the plane
↳ hold that thought!

§10.4 Cross Product

<u>def'n</u>: $\vec{u} \times \vec{v} = \begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$

Mnemonic $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ "matrix" determinant

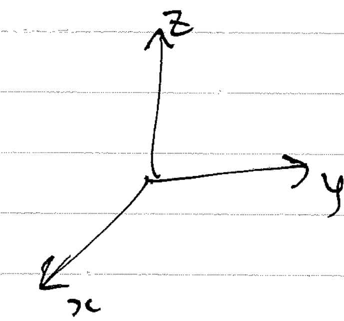
<u>Eg</u> $\vec{i} \times \vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0\vec{i} - 0\vec{j} + 1\vec{k}$

$= \vec{k}$

Similarly: $\vec{j} \times \vec{k} = \vec{i}$, $\vec{k} \times \vec{i} = \vec{j}$

<u>Properties</u>

— Right-hand rule



① * $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$ anticommutative

② $(\vec{u}+\vec{v}) \times \vec{w} = \vec{u}\times\vec{w} + \vec{v}\times\vec{w}$

$\vec{u}\times(\vec{v}+\vec{w}) = \vec{u}\times\vec{v} + \vec{u}\times\vec{w}$

③ $c(\vec{u}\times\vec{w}) = (c\vec{u})\times\vec{w} = \vec{u}\times(c\vec{w})$

④* orthogonality $\vec{u}\times\vec{v} \perp \vec{u}$ and

$\vec{u}\times\vec{v} \perp \vec{v}$ (try with dot)

⑤ $\vec{u}\times\vec{u} = \vec{0}$

⑥ $\vec{u}\times\vec{0} = \vec{0}$

⑦* $\vec{u}\cdot(\vec{v}\times\vec{w}) = (\vec{u}\times\vec{v})\cdot\vec{w}$ Triple Scalar Product.

Geometry: volume of a parallelopiped text pg 599

$$\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

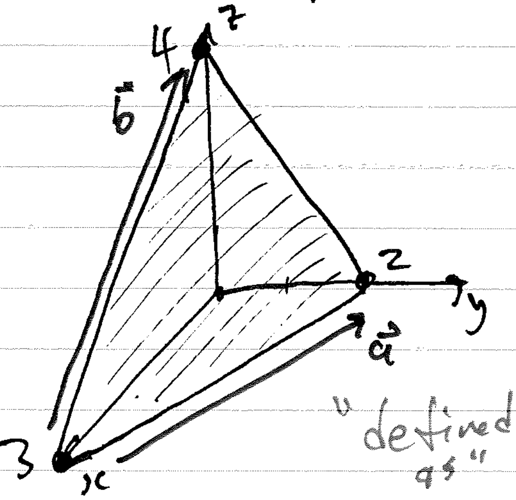
Angles and the cross product

$$|\vec{u} \times \vec{v}| = |\vec{u}|\,|\vec{v}| \sin\theta$$

From this, $\vec{u} \times \vec{v} = \vec{0}$ if and only if $\vec{u}$ and $\vec{v}$ are parallel.

↳ see text for $\vec{u} = 0$ case.

Ex find a unit vector $\vec{n}$ normal to a plane thru $(3,0,0)$, $(0,2,0)$, $(0,0,4)$



$$\vec{a} = \langle 0,2,0\rangle - \langle 3,0,0\rangle = \langle -3, 2, 0\rangle$$

$$\vec{b} = \langle -3, 0, 4\rangle$$

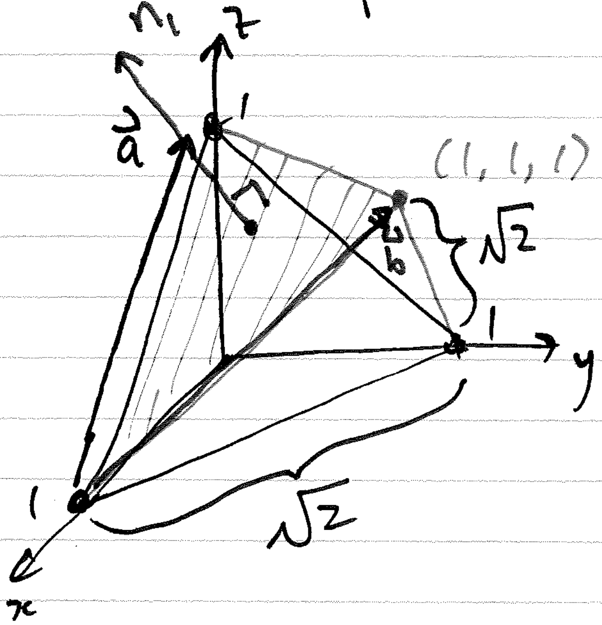
$$\vec{u} := \text{cross}(\vec{a}, \vec{b}) = \dots = \langle 8, 12, 6\rangle$$

"defined as"

$$\vec{n} := \frac{\vec{u}}{|\vec{u}|} = \dots = \frac{\langle 4, 6, 3\rangle}{\sqrt{61}}$$

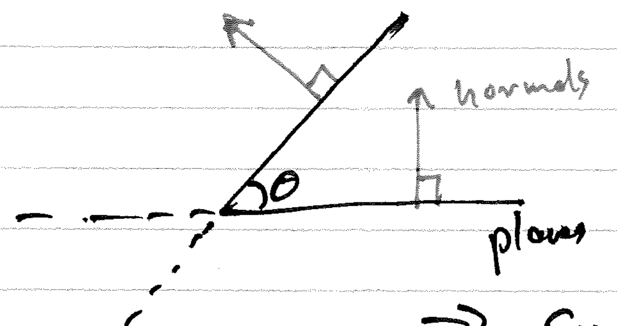
did these calculations in Matlab/Octave

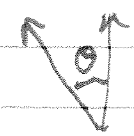
<u>Example</u>: Find the angles b/w the faces of a regular tetrahedron.



① check all 4 Δs have same side lengths

② Insteadd, find angles b/w <u>normals</u> of <u>faces</u>





... $\Rightarrow$ same angle (!)

<u>Take</u>: $\vec{a} := \langle -1, 0, 1 \rangle$, $\vec{b} := \cancel{\langle -1, 0, 1 \rangle}$ $\langle 0, 1, 1 \rangle$

$$\vec{n}_1 = \vec{b} \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = \ldots$$

$$= \langle 1, -1, 1 \rangle$$

Next day, find $\vec{n}_2$, etc.