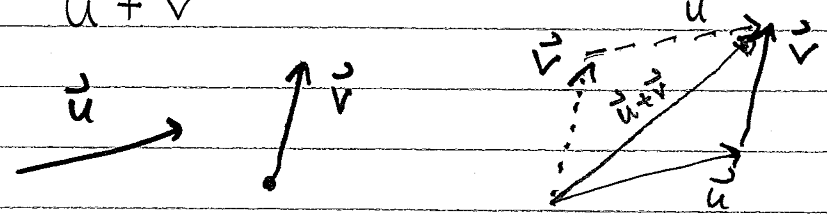
# Math 253, Section 101

- Webwork, HW1 online, due next week.
- Our course does not use connect
  ↳ canvas.ubc.ca
- course website password

## Last day: vectors §10.2

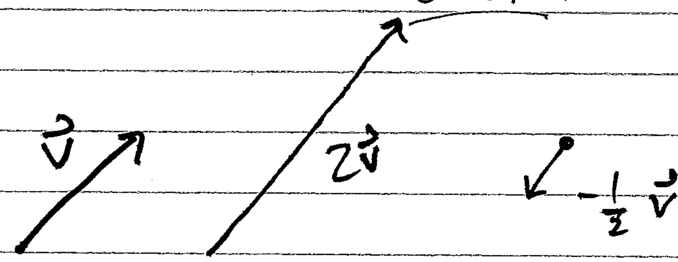
$\vec{u} + \vec{v}$




Defn $c\vec{v}$ has magnitude : $|c| \cdot$ length of $\vec{v}$

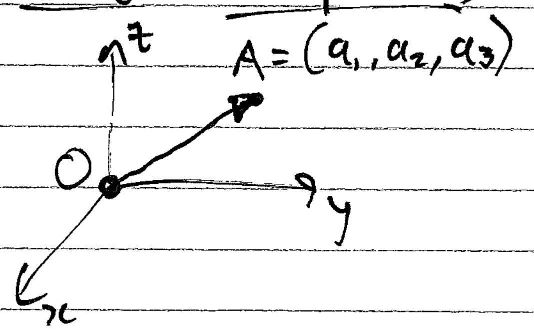
Scalar   vector   and direction : $\begin{cases} \vec{v} & \text{if } c>0 \\ -\vec{v} & \text{if } c<0 \end{cases}$

Eg




## Vector components

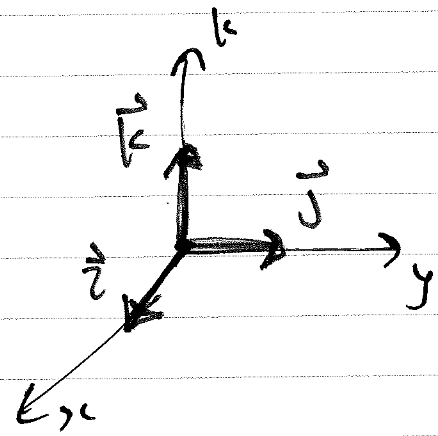
$\vec{a} = \overrightarrow{OA} = \langle a_1, a_2, a_3 \rangle$




$$= \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Standard basis vectors

$\vec{i} = \langle 1, 0, 0 \rangle$
$\vec{j} = \langle 0, 1, 0 \rangle$
$\vec{k} = \langle 0, 0, 1 \rangle$




$$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$$

<u>Vector addition</u> add components

Eg. in 2D $\quad \langle 1, 2 \rangle + \langle 3, 4 \rangle$

$$= (1\vec{i} + 2\vec{j}) + (3\vec{i} + 4\vec{j})$$

$$= (1+3)\vec{i} + (2+4)\vec{j}$$

$$= 4\vec{i} + 6\vec{j} = \langle 4, 6 \rangle$$

Similarly: $c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$

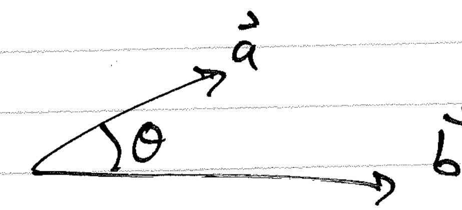
<u>Magnitude</u>: $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

# §10.3 Dot Product

scalar!

**Def'n 1** : $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

**Def'n 2** : $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$




(these are equivalent)

**Notes**: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$, $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$,

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

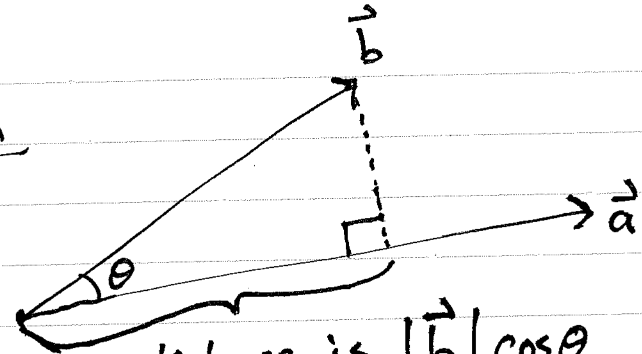
etc (list in text)

**Def'n**: $\vec{a}$ and $\vec{b}$ are <u>orthogonal</u> if the angle b/w them is $\theta = \pi/2$.

$\Rightarrow$ $\vec{a}$, $\vec{b}$ orthogonal if and only if $\vec{a} \cdot \vec{b} = 0$

Note: $\vec{0}$ is orthogonal to any vector

# Projection




distance is $|\vec{b}|\cos\theta$

used def'n 2

def'n

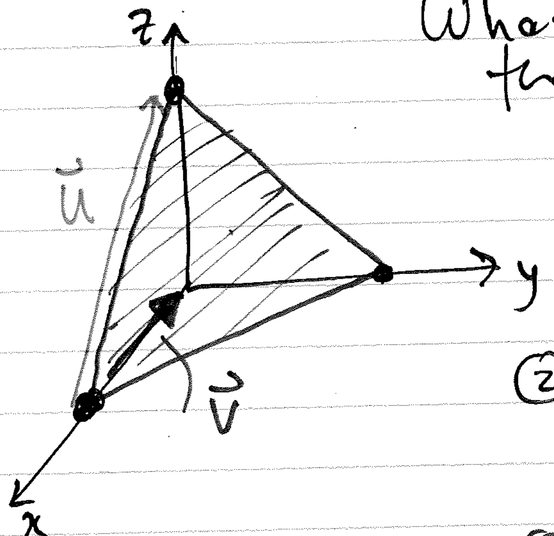
$$\text{proj}_{\vec{a}}\vec{b} = |\vec{b}|\cos\theta \frac{\vec{a}}{|\vec{a}|} = \boxed{\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}} \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$$

vector projection of $\vec{b}$ onto $\vec{a}$

Unit vector in dir $\vec{a}$

scalar projection of $\vec{b}$ onto $\vec{a}$

Ex Suppose we have plane thru $(1,0,0)$, $(0,1,0)$, $(0,0,1)$.
What is the distance b/w the plane and the origin?




① choose any pt on the plane, $(1,0,0)$

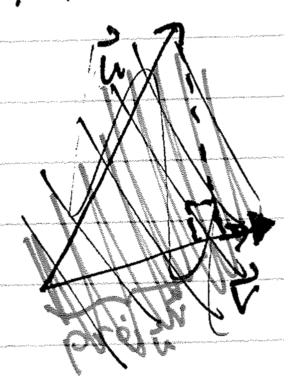
② find a vector from that pt in the plane
$\vec{u} = \langle -1, 0, 1 \rangle$

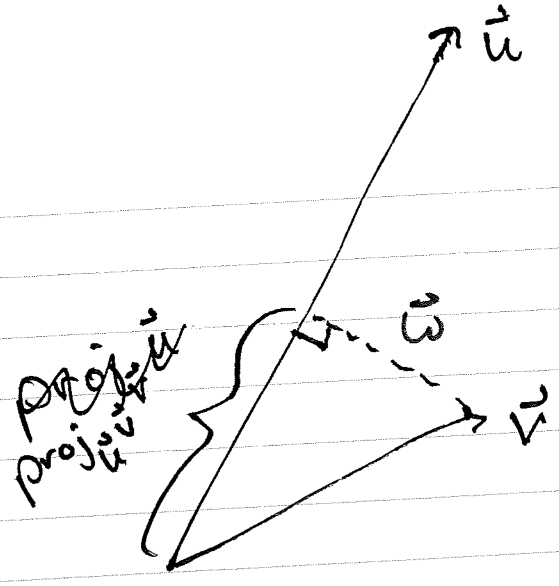
③ let $\vec{v}$ be the vector from pt to origin
$\vec{v} = \langle -1, 0, 0 \rangle$

Note. $|\vec{v}|$ is not the minimum distance

$$\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{v} \cdot \vec{u}}{|\vec{u}|^2} \vec{u} = \frac{((-1)(-1)+0+0)}{(\sqrt{2})^2} \langle -1, 0, 1 \rangle$$
$$= \frac{1}{2} \langle -1, 0, 1 \rangle = \langle -\tfrac{1}{2}, 0, \tfrac{1}{2} \rangle$$

Note: $|\text{proj}_{\vec{u}} \vec{v}|$ is not the minimum dist.






$$\vec{v} = \underbrace{\text{proj}_{\vec u}\vec v}_{\parallel \text{ to } \vec u} + \underbrace{(\vec v - \text{proj}_{\vec u}\vec v)}_{\perp \text{ to } \vec u}$$

$$\vec{w} = \vec{v} - \text{proj}_{\vec u}\vec v = \langle -1, 0, 0\rangle - \langle -\tfrac{1}{2}, 0, \tfrac{1}{2}\rangle$$

$$= \langle -\tfrac{1}{2}, 0, -\tfrac{1}{2}\rangle$$

$$\cancel{|\vec{w}| = \frac{1}{\sqrt{4}}} \qquad |\vec{w}| = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

Q: is that $|\vec{w}| = \cancel{\frac{1}{\sqrt{4}}}\ \frac{1}{\sqrt{2}}$ the minimum distance?

Ex: choose diff pt, try calc again...

for next day..