“One fish,
Two fish,
Red fish,
Blue fish”

Modeling a Fresh Fish Detector

PIMS Modeling Group 1

Simon Fraser University, Canada

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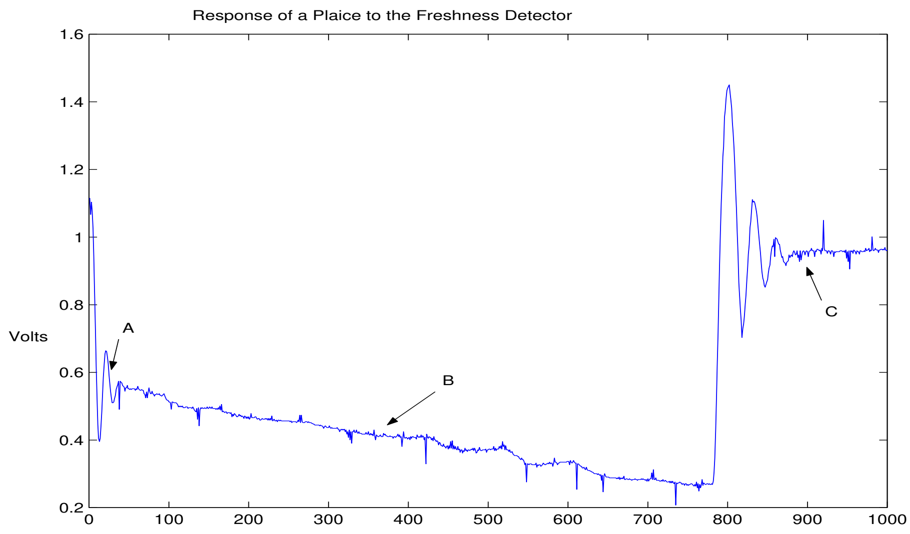


Presentation Outline

- Overview of model
- Digital Signal Processing: De-noising the raw data
- Analysis of distinct phases
  - Phase B
  - Phase A
  - Phase C
- Conclusions

Overview of Model

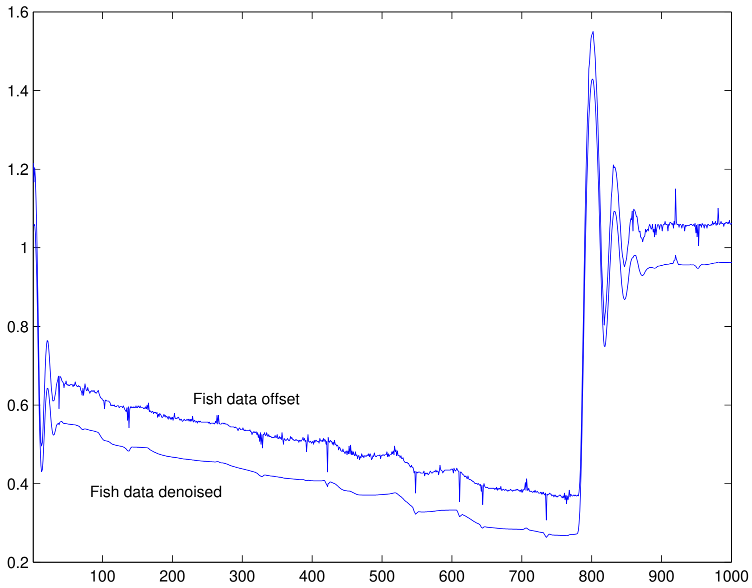
This project models the dynamics of a device for measuring fish freshness. A needle–like probe applies a constant force to the surface of a fish and is then released. The dynamics of the fish–probe system change during the experiment, giving rise to three distinct phases:

- **Phase A**: The response resembles an oscillating decaying exponential function, which might be modeled as a mass–spring system with damping.
- **Phase B**: A linearly (or exponentially) decaying response is observed.
- **Phase C**: A second oscillating decaying exponential behavior.

The approach involved modeling the separate phases as independent processes. These were then combined to produce a working model for the entire fish–probe system.

De-noising the Raw Data

- **Why De-noising?**
  1. The signal contains noise.
  2. Increase the modeling accuracy by increasing the SNR.
  3. Numerical computation of the velocity and acceleration of the probe.
- **De-noising methods:**
  - FFT approach
  - Wavelet approach $\leftarrow$ Better!



Phase A

We attempted to model the fish skin as a visco-elastic system described by a damped linear oscillator:

$$M\ddot{x} + \beta\dot{x} + \alpha x + Mg = 0, \tag{1}$$

where $M = 10$ grams (mass of probe), $g =$gravity, and $x = x(t)$ is the position of the probe. With constant coefficients, the solution to Equation (1) is:

$$x(t) = e^{-\delta t}[A\cos(\omega t) + B\sin(\omega t)] + D \tag{2}$$

where

$$\omega = \sqrt{\frac{\alpha}{M} - \delta^2} \quad \text{and} \quad \delta = \frac{\beta}{2M}$$

Also, D represents a vertical shift. We then fit this functional form to the data using a nonlinear least squares algorithm (Matlab). The results show that Phase A can indeed be modeled by a damped linear oscillator.

Phase B

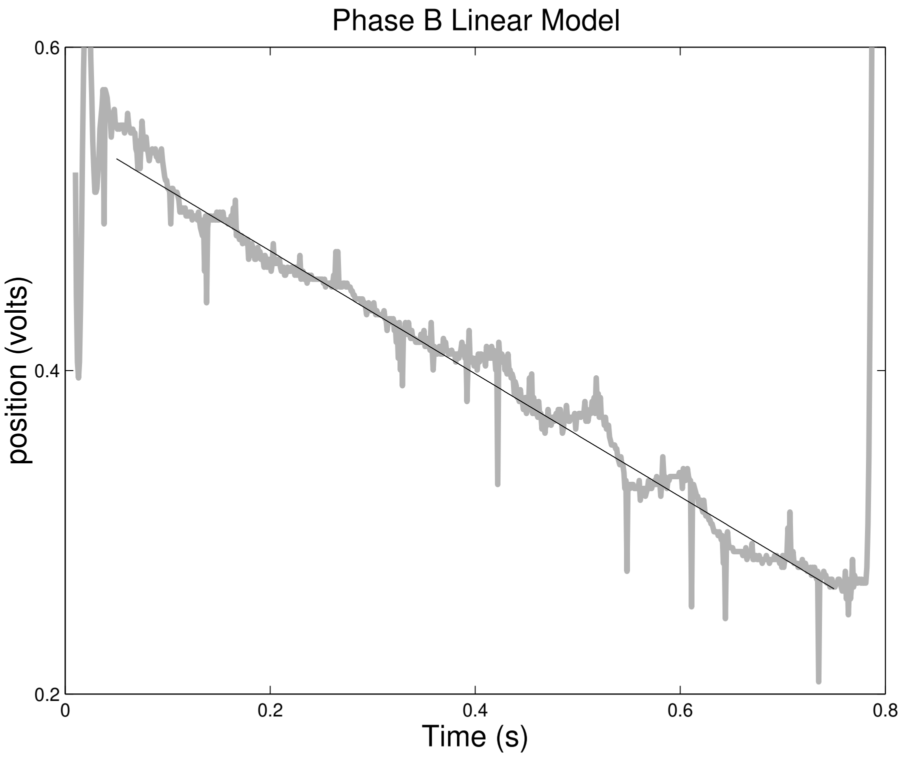
We model Phase B using the linear model

$$\boldsymbol{\alpha \dot{x} = F}, \tag{3}$$

derived from *Darcy's Law* relating the *velocity* of the probe to the constant *pressure gradient* resulting from the *migration of the fluid* away from the probe.
This ODE has solution

$$\boldsymbol{x(t) = X_0 + \frac{F}{\alpha} t} \tag{4}$$

and we fit the parameters $\boldsymbol{X_0}$ and $\boldsymbol{\frac{F}{\alpha}}$.



Phase C

Phase C also indicates damped oscillations.

- A damped linear oscillator was first implemented but did not fit the data.

- We considered a damped oscillator with nonlinear damping and restoring coefficients:

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \boldsymbol{x} + \boldsymbol{\alpha}_2 \boldsymbol{x}^2 \quad \text{and} \quad \boldsymbol{\beta} = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \dot{\boldsymbol{x}} + \boldsymbol{\beta}_2 \dot{\boldsymbol{x}}^2$$

- The equation was solved using Matlab, then the nonlinear least squares routine fit the resulting solution to the original fish data. The Figure reveals that a damped nonlinear oscillator cannot accurately describe the dynamics of Phase C.

- For some insight we turn to the acceleration data obtained earlier.

Phase C

From the graph we can see:

- A section where the probe is in free fall.
- Therefore we propose a loss of contact model where the probe and fish skin separate.
- There are $\mathbf{3}$ distinct sections:
  - Section I — The force is released and the probe and skin move together in contact.
  - Section II — The probe and skin lose contact and move separately.
  - Section III — The probe and skin come back into contact and move together.

Model the fish–probe system as an impact oscillator

- Sections I and III can be modeled with the same dynamics
  - With a damped linear oscillator:

    $$(M+m)\ddot{x} + \beta\dot{x} + \alpha + (M+m)g = 0 \quad (5)$$

    where m = mass of fish skin in contact with the probe.

  - The oscillatory behavior is interrupted by Section II.
  - The solution in Section III is simply shifted in time with slight change in amplitude.
- In Section II the fish–probe system is uncoupled; probe is in free fall while the skin continues to oscillate.
- We used continuity in position and velocity to connect Sections I and II.
- The data fit well indicating an accurate model.