Plan for today

- Integrating Symmetric functions
- Qualitative analysis of integrals
- Areas between curves
- Workshop 2

Midterm is next Wed, Jan 30

at 7pm in Angus 098.

- Slight changes to how the software will work
  - You won't have "submit quiz" on canvas to exist.
  - You will have to "grade test" exactly once.

Midterm covers Ch 1-4 (HW 1-4)

To prepare:

- Finish HW assignments
- Read the book
- Practice problems from book and syllabus
Symmetry (3.6)

Even functions are symmetric about the y-axis (reflection)

\[ f(x) = f(-x) \]

\[ ax: x^2 \cos(x) \]

Any even polynomial (exponent is even for all terms)

Odd functions are symmetric about a rotation around the origin

\[ f(x) = -f(-x) \]

\[ ax: x^3 \sin(x) \]

Any odd polynomial

Every odd function has \( f(0) = 0 \).
• The derivative of an even function is odd

• The derivative of an odd function is even: \( f'(x) = \frac{d}{dx} f(-x) \) 
  \[ = (-1) f'(-x) \] 
  \[ = f'(-x) \]

• The integral of an even function is odd (up to a constant) \( [f F(0) = 0] \)
  More precisely, \( F(x) = \int_{-a}^{x} f(t)dt \) is odd

• The integral of an odd function is even.

If \( f \) is even, \( \int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx \)

If \( f \) is odd, \( \int_{-a}^{a} f(x)dx = 0 \)
Ex: For \( R > 0 \), calculate

\[
\int_{-R/2}^{R/2} \frac{x}{\sqrt{R^2 - x^2}} \, dx
\]

\[f(x) = \frac{x}{\sqrt{R^2 - x^2}}\]

\[f(-x) = \frac{-x}{\sqrt{R^2 - (-x)^2}} = -\left(\frac{x}{\sqrt{R^2 - x^2}}\right) = -f(x)\]

\(f\) is odd, the bounds of integration are symmetric, so the integral is 0.

Products:

\[h(x) = f(x)g(x)\]

- Even times even is even
- Odd times even is odd
- Odd times odd is odd

\[\cos(x) \cdot \sin(x)^3\]

- \(\sin(x)\) is odd times odd so is \(\text{even}\)
- \(\sin(x) \sin^2(x)\) is odd times even so is \(\text{odd}\)
- \(\cos(x) \sin^3(x)\) is even times odd so is \(\text{odd}\)
\[ \int_{-\pi}^{\pi} \cos(x) \sin^3(x) \, dx = 0 \quad \checkmark \]

\[ \int_{0}^{\pi} \sin(x) \, dx \]

\[ \int_{-\pi}^{\pi} -\sin(x) \, dx = 0 \]
Qualitative Analysis

\[ f(x) = x^3 - x \]

\[ F(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 \]

\( \text{When } f \text{ is positive, } F \text{ is increasing.} \)

\( \text{When } F \text{ has a max or min, } f = 0 \)

\( f \text{ goes from positive to negative} \)

\( \text{If } \frac{dF}{dx} > 0 \text{ then } F \text{ is concave up if } f \text{ is increasing} \)

\( \text{If } \frac{dF}{dx} < 0 \text{ then } F \text{ is concave down if } f \text{ is decreasing} \)
Area between curves

\[ f(x) = x^3 \quad \text{and} \quad g(x) = x \]

Find the lower and upper bounds (where \( f = g \))

\[ \text{Area} = \int_{-1}^{1} |f(x) - g(x)| \, dx \]

Find intervals where \( f > g \) and \( g > f \)

\[ = \int_{-1}^{0} (f(x) - g(x)) \, dx + \int_{0}^{1} (g(x) - f(x)) \, dx \]

Symmetry

\[ = 2 \int_{0}^{1} (x - x^3) \, dx = 2 \left[ \frac{1}{2} x^2 - \frac{1}{4} x^4 \right]_{0}^{1/2} \]

\[ = 2 \left[ \frac{1}{2} \cdot \frac{1}{4} - \frac{1}{4} \cdot \frac{1}{16} \right] - 2 \left[ 0 - 0 \right] \]

\[ = \frac{1}{12} \]
1) Mass produced is \( \int (f(s) - g(s)) \, ds \)

2) Peak occurs when \( f = g \) at \( t = \frac{3}{4} \)

3) Calculate the integrals as a practice exercise at home