**Problem 1.** Fix an infinite graph $G$, with an exhaustion $G_n$ (growing sequence of finite subsets so that every vertex is eventually contained in $G_n$). Prove (as claimed in class) that $\lim R_{\text{eff}}(a, G_n^c)$ does not depend on the choice of exhaustion.

**Problem 2.** For a finite graph $G$ with vertices $a, z$, prove that $R_{\text{eff}}(a, z)$ is a continuous function of the edge conductances.

**Problem 3.** Two graphs $G, H$ are said to be roughly isometric if there is a function $f : G \to H$ so that for some constants $a, b, c$,

$$a^{-1}d(x,y) - b \leq d(f(x), f(y)) \leq ad(x,y) + b,$$

and such that every $z \in H$ is at distance at most $c$ from $f(x)$ for some $x \in G$.

(a) Prove that the 3-regular tree is roughly isometric to the 4-regular tree.

(b) Prove that $\mathbb{Z}^d$ is not roughly isomorphic to $\mathbb{Z}^{d'}$ if $d \neq d'$. Prove

**Problem 4.** Show that every Markov chain on the 3-regular tree (i.e. $p(x, y) > 0$ only for $x \sim y$) is reversible.

**Problem 5.** Recall that a Markov chain is positive recurrent if the return times have positive expectation: $\mathbb{E}_a \tau_a^+ < \infty$. Prove that a reversible Markov chain is positive recurrent if and only if $\sum_{x \sim y} c(x, y) < \infty$. 
