Kuhn poker:

* Deal cards, ante 1 each
* Player 1 (bets 1) or (waits) B or C
* Player 2 (bet 1) or (not).
* If I waited, player 2 bets,
then P1 can call or fold.
Notice that these strategies contain interesting behaviors such as bluffing (for example, player 2 always bets holding a 2 if player 1 passes) and slow-playing (for example, player 1 doesn't always bet on the first round even if he holds an ace).
Dealer: See J Q K
    See Bet or Check
(K, B) \Rightarrow always bet
(J, B) \Rightarrow always fold.

strategy: probability for each action in each
case that can occur.
Finding equilibria

0-sum: \( \max_x \min_y x^T Ay \)

\[ \Rightarrow \text{pivot algorithm.} \]

General sum: eliminate strictly dominated strategies: if a row is strictly dominated remove it.

Principle of indifference:

If \( S_x = \text{set of} \{ i : x_i > 0 \} \)

\( S_y = \{ i : y_i > 0 \} \)

If \((x, y)\) is an equil., all \( i \in S_x \) equally good, and better or equal to \( j \notin S_x \)

\( A_y \) has equal entries in \( S_x \)

\( x^T B \) has equal entries in \( S_y \)

e.g.: \( x = (\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, 0, 0, \frac{1}{3}) \) \( S_x = \{1, 2, 3, 6\} \)
Bonus problem: Find N.E. for smallest number game with n players.

Glimpse of evolutionary game theory:
Assume general population uses strategy \( x \).

Payoff: \( x^T A x \)

Let new mutation have strategy \( z \):

\[
\begin{cases}
\text{If } z^T A x > x^T A x : z \text{ beneficial.} \\
\Rightarrow x \text{ unstable.}
\end{cases}
\]

\[
\begin{cases}
\text{If } z^T A x = x^T A x \text{ but } z^T A z > x^T A x \\
\Rightarrow x \text{ unstable.}
\end{cases}
\]

If no such \( z \) then we call \( x \) evolutionarily stable.
Aggressive: Hawk

Not: Dove

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<tr>
<td>D</td>
<td>(4,4)</td>
<td>(0,8)</td>
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<tr>
<td>H</td>
<td>(8,0)</td>
<td>(5,5)</td>
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If $s > 0$: strict domination shows that only NE is $(H, H)$.

If $s < 0$: mixed NE $(p, 1-p)$

$$4p + 8(1-p) = s(1-p) \Rightarrow 4p + (8-s)(1-p) = 0$$

$$p = \frac{8-s}{4-s} \quad \text{payoff is } s(1-p) = \frac{-4s}{4-s}$$

This is a stable equil.
RPS

\[
\begin{pmatrix}
R & P & S \\
R & 0 & 1 & 1 \\
P & 1 & 0 & -1 \\
S & -1 & 1 & 0
\end{pmatrix}
\]

\(\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)\) gives value of 0.