**Problem 1.** Find the NTU-solution and the equilibrium exchange rate of the following game without a fixed threat point.

\[
\begin{pmatrix}
(4,8) & (6,6) \\
(6,0) & (0,8)
\end{pmatrix}
\]

**Problem 2.** (a) Let \(v_1\) and \(v_2\) be two games in coalition form, and suppose both have a non-empty core. Prove that for any positive \(a, b\), the game \(av_1 + bv_2\) also has a non-empty core.

(b) Give an example of games \(v_1, v_2\) so that \(v_1\) has an empty core but \(v_2\) and \(v_1 + v_2\) have a non-empty core. (c) Give an example of games \(v_1, v_2\) so that \(v_1\) and \(v_2\) have an empty core but \(v_1 + v_2\) has a non-empty core.

**Problem 3.** A toy is sold for $60 and consists of three parts: I, II, III. There is one manufacturer of part I, two manufacturers of part II, and three manufacturers of part III. No part of the toy can be sold separately. Assume the cost of manufacturing the parts is $0.

(a) Describe the problem as a game in coalitional form, i.e. define the characteristic function.  
(b) Compute the core of the game. 
(c) Find the Shapley values.

**Problem 4.** Suppose the value of a set depends only on the size of the set. If there are three players, and \(v(\{1, 2, 3\}) = 1\), and for every \(i, j\) we have

\[
v(\{i\}) = a \quad v(\{i, j\}) = b.
\]

(a) Find the Shapley value of this game. 
(b) For which \(a, b\) is the core non-empty?

**Bonus Problem 5.** A coalition game is called **convex** if for any sets \(S, T\) we have

\[
v(S \cup T) + v(S \cap T) \geq v(S) + v(T).
\]

Prove that a convex game has a non-empty core, and that the Shapley value is in the core.