Recall (Statistics)

Sample mean: \( \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \)

Sample Variance: \( S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \)

\[ \mu = E X, \quad \sigma^2 = Var(X) \]

LLN: \( \bar{X} \to \mu \) as \( n \to \infty \)

\( S^2 \to \sigma^2 \)

CLT: \( \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \to N(0,1) \) as \( n \to \infty \)

If Hypothesis is \( X \) has \( E(X) = \mu \), \( Var(X) = \sigma^2 \)

Let \( z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \) (z-score)

E.g. Sum of 100 dice is 3412, \( \bar{X} = 3.412 \)
P-score: \[ P\left( |N(0,1)| > 1.21 \right) \]

e.g. \( z = 1.3 \) \( \Rightarrow P( |N(0,1)| > 1.3 ) \approx 0.19 \ldots \)

If \( \sigma \) not known, \( n \approx 1 \) then \( \bar{X} \approx \mu \)

Use \( \frac{\bar{X} - \mu}{s/\sqrt{n}} \) instead of \( \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \)

Note: In this case, if \( n \) small, \( \frac{\bar{X} - \mu}{s/\sqrt{n}} \) has Student-T distribution pdf \( c(1 + t^2_{n-1})^{-n/2} \) \( \approx e^{-t^2/2} \) if \( n \approx 1 \)

Confidence intervals: \( [\bar{X} - \alpha, \bar{X} + \alpha] \) s.t. it contains \( \mu \) w.h.p.

E.g. 95% C.I.: take \( \alpha \) s.t. \( P( |X - \mu| > \alpha ) < 0.05 \)

By CLT, this \( \alpha = \frac{\sigma}{\sqrt{n}} \cdot 1.96 \)
e.g. opinion poll: 600 of 1000 people say Yes.

Estimate $\sigma^2 = \text{Var}(\text{Bern}(0.6)) = (0.6)(0.4) = 0.24$

$\alpha = \frac{\sigma}{\sqrt{1000}} = \sqrt{0.00024} \cdot 1.96 \approx 0.03$.\_
Start Gamblers Ruin

Start with $k \in \{0, 1, \ldots, n\}$, bet 1 $ on a fair coin.
Stop when reach either 0 or n.

Q: What is the probability that Mark reaches 0 before n?
Let \( q(k) = P( \text{reach } n \text{ before } 0, \text{ starting at } k) \).

We will find \( q(0), q(1), \ldots, q(n) \).

**Key:**
\[
q'(k) = P(\text{win} | \text{win 1st toss}) \cdot \frac{1}{2} + P(\text{win} | \text{lose 1st toss}) \cdot \frac{1}{2}
\]

\[
q(k+1) = \frac{1}{2} q(k+1) + \frac{1}{2} q(k-1)
\]

for \( k = 1, \ldots, n-1 \).

Simplify:
\[
q(k+1) - q(k) = q(k) - q(k-1)
\]

Thus \( q(k) = A + Bk \) for some \( A, B \).

\[
q(0) = 0, \quad q(n) = 1
\]

\[
A + B \cdot 0 = 0, \quad A + Bn = 1
\]

\[
A = 0, \quad B = \frac{1}{n}
\]
This is called a **Simple Random Walk** started at $K$. This a **stochastic process** $X_t = \text{money at time } t$.

What if win each coin w.p $p < \frac{1}{2}$

[Diagram showing a stochastic process with transitions $1-p$ and $p$ between states $K$, $K+1$ and $K-1$.]

Get eqn.

$q(K) = p(q(K+1)) + (1-p)q(K-1)$

**Example**: Roulette:
- 18 Red
- 18 Black
- 2 Green

Bet on Red: $p = \frac{18}{38} = \frac{9}{19}$