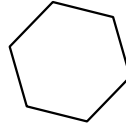Blitz, Page 1

1. What is the value of 15% of 40?

1. _____

2. How many diagonals does a regular hexagon have? (A diagonal is any line segment that joins two vertices and is *not* an edge.)

2. _____ diagonals

3. For a school fundraiser, Dave sells chocolate bars. He wants to raise at least $100. He sells each chocolate bar for $1.25. What is the minimum number of bars he needs to sell?

3. _____ bars

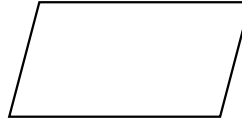4. Let $F(x) = x^2 + 1$. What is the value of $F(F(1))$?

4. _____

5. Round the following sum to the nearest whole number:

$$\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5}.$$

5. _____

6. Four fair coins are tossed. What is the probability of getting an even number of heads? Express the answer as a common fraction.

6. _____

7. If you run at a speed of 12 km/hour for 20 minutes, and then at a speed of 10 km/hour for 45 minutes, what is the total distance, in km, that you ran? Express the answer as a decimal, correct to 1 decimal place.

7. _____ km

8. In a certain parallelogram, the degree measure of one of the internal angles is five-sevenths of the degree measure of another internal angle. How many degrees are in the measure of one of the smaller internal angles of the parallelogram?

8. _____ degrees

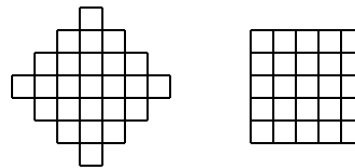9. Simplify: $\dfrac{\sqrt{9 + \dfrac{1}{7}}}{\sqrt{\dfrac{1}{7}}}$

9. _____

10. What is the largest prime factor of 2012?

10. _____

11. Four hundred and forty students took part in a Math Challengers contest. Each student competed in the grade 8 category or the grade 9 category, but not in both. If 198 students were in the grade 9 category, how many percent of the students competed in the grade 8 category?

11. _____ percent

12. Each of the two figures below is made up using twenty-five 1×1 squares. What is the positive difference between the perimeters of the two figures?

12. _____ units

13. Express
$$\frac{1 + 3 + 3^2}{1 + 3 + 3^2 + 3^3 + 3^4 + 3^5}$$
as a common fraction.

13. _____

14. Alicia has 5 dollars less than Beti, and Cecille has as much money as Alicia and Beti have between them. Altogether, the three people have a total of 270 dollars. How many dollars does Alicia have?
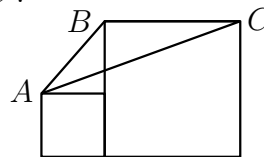
14. _____ dollars

15. What is the value of the mean of the numbers in the following list?   15. _____
Express the answer as a common fraction.

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6$$

16. Alan left home to go to the mall. He spent \$2.50 for the Skytrain trip.   16. _____ dollars
Then he spent six-sevenths of the money he had left on t-shirts. Then
he spent two-thirds of the money he had left after buying the t-shirts
on a snack. After that, all he had left was \$2.50 for the Skytrain trip
home. How many dollars did Alan have immediately before he left
home to go to the mall?

17. There are 12 jelly beans on a tray, 6 blue and 6 yellow. Alphonse   17. _____
eats 2 of the jelly beans, chosen at random. What is the probability
that these 2 jelly beans are of *different* colours? Express the answer
as a common fraction.

18. In the picture below, the smaller square has side 7 and the larger   18. _____ units^2
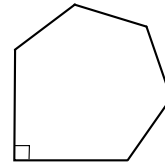square has side 15. What is the area of $\triangle ABC$?

19. Call the integer n *special* if the sum of the decimal digits of n is 12.   19. _____ squares
How many of the *perfect squares* from 1 to 400 are special?

20. Express $\dfrac{\sqrt{75} - \sqrt{3}}{\sqrt{75} + \sqrt{3}}$ as a common fraction.   20. _____

21. A convex hexagon has one internal right angle. The other five internal angles are all equal to each other. How many degrees are in the measure of one of these five angles?

21. _____ degrees

22. How many integers between 1 and 1000 are divisible by 3 but not by 9?

22. _____ integers

23. If $x^{2012} = 9$, what is the value of x^{3018}?

23. _____

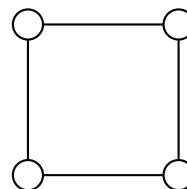24. What is the smallest positive integer n such that

$$1 + 2 + 3 + \cdots + (n - 1) + n$$
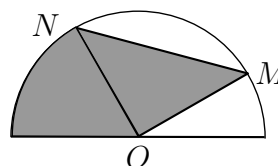
is a multiple of 100?

24. _____

25. How many ways are there to colour the four circles below, using no colours other than blue, red, or yellow (only one colour to each circle), so that any two circles directly joined by a line are of different colours?

25. _____ ways

26. A half-circle has centre at the origin $O(0,0)$. Points $M(\sqrt{75}, 5)$ and $N(-5, \sqrt{75})$ are on the half-circle as shown. What is the area of the shaded region, correct to the nearest integer?
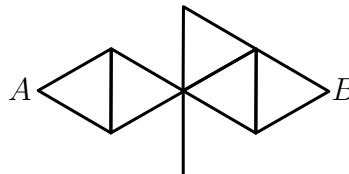
26. _____ units^2

Bull's-eye, Page 1: Problem Solving

1. An exam consists of multiple choice questions, worth 2 marks each, and "short answer" questions, worth 4 marks each. The total number of questions on the exam is 39, and the maximum mark obtainable is 100. How many multiple choice questions are on the exam?

   1. _____ questions

2. A large group had a banquet. The food bill was $1000. In addition to that, the group had to pay the 12% HST. The group decided to leave a tip of 15% of the combined cost of food and HST. What was the amount, in dollars, of the tip?

   2. _____ dollars

3. Alicia has exactly $9.99 in standard Canadian coins. (The standard Canadian coins are 1 cent, 5 cents, 10 cents, 25 cents, $1, and $2). What is the smallest number of coins that Alicia could have?

   3. _____ coins

4. The lines below represent the streets of a town. Each side of every small triangle has length 1, and the additional segment has length 1. You want to walk from point A to point B, and travel at least once along each street of the town. What is the shortest possible length of the path?
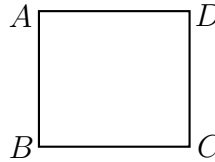
   4. _____ units

Bull's-eye, Page 2: Numbers and Combinatorics

5. What is the smallest positive integer which can be expressed as the sum of 6 consecutive multiples of 6, some of which could be 0 or negative?

5. _____

6. A box contains 9 marbles, of which 4 are white, 3 are black, and 2 are blue. Dina removes 2 randomly chosen marbles from the box, and places them on a table. What is the probability that the 2 marbles are of the same colour? Express the answer as a common fraction.

6. _____

7. Call the integer n *good* if $n > 0$ and 2 divides n, 3 divides $n+1$, 4 divides $n+2$, and 5 divides $n+3$. The smallest good integer is 2. What is the next good integer?

7. _____

8. Let M be a three-digit number, with all digits different from 0 (so 234 and 665 are of the right kind, but 302 is not). Let N be a three-digit number obtained by changing the order of the digits of M. Given that $M + N = 949$, what is the largest possible value of N?

8. _____

Bull's-eye, Page 3: Geometry

9. The area of rectangle $ABCD$ is $\frac{5}{8}$ units^2. Side AB has length $\frac{3}{4}$ units. What is the length of side BC? Express the answer as a common fraction.

   9. _____ units

   ```
   A ┌─────────┐ D
     │         │
     │         │
   B └─────────┘ C
   ```
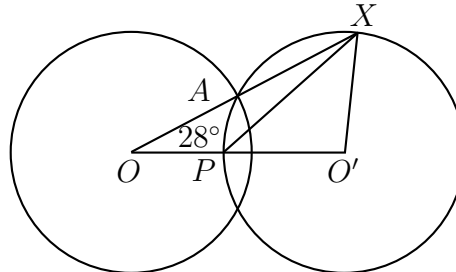
10. Three gold spheres have, respectively, diameter 9 cm, 12 cm, and 15 cm. They are melted down and made into a single gold sphere. What is the diameter of that sphere?
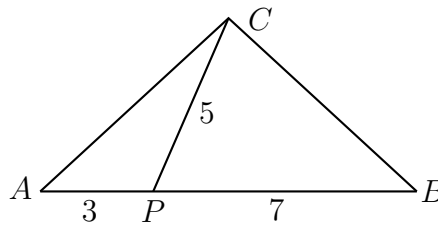
    10. _____ cm

11. The two circles below have the same radius, and have centres O and O'. The line segment joining O and O' meets the circle with centre O' at the point P. Point A is one of the intersection points of the circles, and the line OA meets the circle with centre O' at a second point X. Given that $\angle AOP$ is 28 degrees, how many degrees are in $\angle XPO'$?

    11. _____ degrees

12. Triangle ABC is isosceles, with $CA = CB$. Point P is on AB. Given that $AP = 3$, $PB = 7$, and $CP = 5$, what is the area of $\triangle ABC$? Express the answer in simplest radical form.
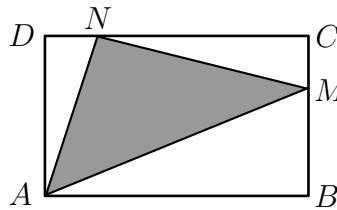
    12. _____ units^2

1. What is the value of

    1. _____

$$(1 \times 1!) + (2 \times 2!) + (3 \times 3!) + (4 \times 4!) + (5 \times 5!)?$$

2. A large group had a banquet. The total bill was for $1000.15. This amount was for the food *plus* the HST: the 12% HST was included in the $1000.15 bill. The group decided to leave in addition a tip of 15% on the cost of the food only, not the HST. What was the total amount, in dollars, paid by the group, including the tip? Express the answer as a decimal, to the nearest cent..

    2. _____ dollars

3. The figure $ABCD$ is a rectangle. The length of BM is $\frac{2}{3}$ times the length of side BC, and the length of CN is $\frac{4}{5}$ times the length of side CD. What is the ratio of the area of the shaded triangle to the area of rectangle $ABCD$? Express the answer as a common fraction.

    3. _____

4. The function $f(x)$ satisfies the equation

    4. _____

$$f(a+b) = \frac{f(a) + f(b)}{1 - f(a)f(b)}$$

   If $f(a) = \dfrac{1}{2}$ and $f(b) = \dfrac{1}{3}$, what is the value of $f(a+b)$?

5. The number 999,999,999,999,999 is multiplied by 999. How many 9's are there in the decimal representation of the product?

    5. _____ 9's

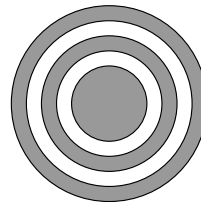Co-op, Page 2: Team answers must be on the *coloured* page.
   Answers on a white page will not be graded.

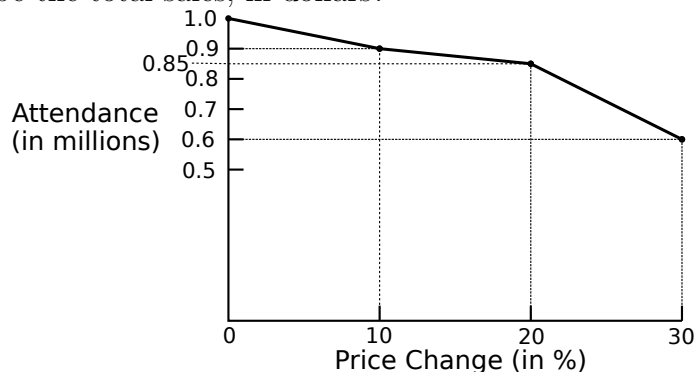6. What is the smallest integer n such that $n(n+2012)(n-2013) > 0$?   6. _____

7. The energy released by an earthquake is measured using the Richter   7. _____
   Scale. An earthquake of magnitude 9 releases 10 times as much en-
   ergy as an earthquake of magnitude 8. In the same way, a quake of
   magnitude 8.2 releases 10 times as much energy as a quake of magni-
   tude 7.2. What is the ratio of the energy released by a magnitude 8.5
   earthquake to the energy released by a magnitude 8.0 earthquake?
   Express the answer as decimal, correct to 2 decimal places.

8. Ten people, Alan and Beti and 8 others, are divided at random into   8. _____
   two groups, one with 4 people and the other with 6 people. What
   is the probability that Alan and Beti end up in the same group?
   Express the answer as a common fraction.

9. The five circles in the picture have the same centre, and their radii   9. _____ percent
   are 2.5, 3.5, 4.5, 5.5, and 6.5. How many percent of the area of the
   largest circle is shaded? Give the answer rounded to the nearest 1
   percent.

10. A poll of hockey fans reveals that if the price per ticket is $100, then   10. _____ dollars
    1 million people will attend over the entire season. If the ticket price
    is increased by a certain percentage, the attendance changes as in
    the graph. If management charges the price that will maximize its
    sales revenue, but definitely at least $100 and no more than $130 per
    ticket, what will be the total sales, in dollars?

11. Suppose that a, b, c, and d are non-negative integers such that

11. _____

$$a + 8b + 64c + 512d = 2012.$$

What is the smallest possible value of $a + b + c + d$?

12. The game of *Canadian checkers* is played on a 12×12 square board divided into 144 unit squares. A circle of radius 6 is drawn on a Canadian checkerboard, with centre the centre of the board. How many of the 144 unit squares have *all* their vertices on or inside the circle? (The unit squares are not shown in the picture.)

12. _____ squares

13. What is the sum of the positive divisors of the sum of the positive divisors of 200? Note that 1 and n are divisors of n.

13. _____

14. Note that $1 + 2 + 3 + \cdots + 7 + 8 = 36$, and 36 is a perfect square. What is the smallest perfect square greater than 36 which is the sum of the first n positive integers for some n?

14. _____

15. Triangle ABC has $AB = 10$ and $AC = 14$. The three heights AR, BQ, and CP are drawn and meet at O. The distance AP is equal to 6. Let $OQ = x$, and draw the circle with centre O and radius x. What is the area of the circle? Express the answer as a decimal, correct to 1 decimal place.

15. _____ units^2