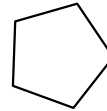Blitz, Page 1

1. Eighty percent of eighty percent of a number is 144. What is the number?

1. _____

2. How many diagonals does a regular pentagon have?

2. _____ diagonals

3. A tiny test consists of 3 multiple choice questions. There are 4 choices of answer for each question, exactly one of which is correct. Alan chooses at random one of the 4 answers to each question. What is the probability that Alan answers all 3 questions correctly? Express the answer as a common fraction.

3. _____

4. A cookie costs $1.20. Dina buys 7 cookies, and hands the cashier a 10 dollar bill. How much money should Dina get back in change? Express the answer in dollars, to the nearest cent. (An answer like 2.70 has the right shape, while 2.7 does not.)

4. _____ dollars

5. What is the whole number which is nearest to $\sqrt{2011}$?

5. _____

6. A cup of flour has 400 Calories. A cup of lard has 1700 Calories. A pie crust is made using two cups of flour, one cup of lard, and nothing else. How many percent of the Calories in the pie crust come from the lard?

6. _____ percent

7. The height of a pyramid is increased by 30%. The base remains unchanged. By how many percent does the volume of the pyramid increase?

7. _____ percent

Blitz, Page 2

8. The sum of the decimal digits of the whole number n is 33. What is the smallest possible value of n?

8. _____

9. Suppose that $f(n+2) = f(n) + f(n+1)$ for every whole number n. Given that $f(1) = 1$ and $f(3) = 48$, what is the value of $f(4)$?

9. _____

10. Alicia tosses 2 fair coins, and then Beti tosses 2 fair coins. What is the probability that they each get the same number of heads? Express the answer as a common fraction.

10. _____

11. Express as a common fraction:

$$\frac{101 + 103 + 105 + \cdots + 197 + 199}{201 + 203 + 205 + \cdots + 297 + 299}.$$

11. _____

12. A rectangle has area 1 square unit. Its width is nine-sixteenths of its length. How many units are in the perimeter of the rectangle? Express the answer as a common fraction.

12. _____ units
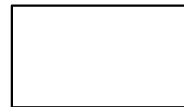
13. What is the smallest positive integer n such that the leftmost decimal digit of 3^n is 6?

13. _____

14. The first term of an arithmetic sequence is 1. The tenth term is 400. What is the fourth term of the sequence?

14. _____

15. What is the smallest possible value of $\left|x - \dfrac{1}{7}\right| + \left|x - \dfrac{1}{6}\right|$ as x travels over the real numbers? Express the answer as a common fraction.

15. _____

16. At a Math Challengers event, 240 people ate *one or more* slices of pizza, 140 ate *two or more* slices, 40 ate three slices, and no one ate more than three. How many slices of pizza were eaten?

16. _____ slices

17. A gambler tosses a fair coin 4 times. She wins if during the tossing she gets 3 or more heads in a row or 3 or more tails in a row. What is the probability that the gambler wins? Write the answer as a common fraction.

17. _____

18. On Thursday, Alphonse began to read a 360 page book. On Friday, he read twice as many pages of the book as he read Thursday. On Saturday, he read twice as many pages as he read Friday. On Sunday, he read twice as many pages as he read Saturday, and finished the book. How many pages of the book did Alphonse read on Thursday?

18. _____ pages

19. A letter E was made by pasting onto a sheet of paper three rectangular 2 cm by 10 cm strips of cardboard next to a rectangular 2 cm by 20 cm strip of cardboard, as in the picture below. No cutting was done. What is the perimeter, in cm, of this letter E?

19. _____ cm
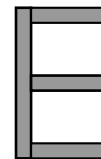
20. Define the number N by

$$N = 1223334444555556666667777777888888889999999999.$$

So the decimal expansion of N is 1 followed by two 2's followed by three 3's followed by four 4's, and so on, until it ends with nine 9's. Alan writes the number N over and over again. What is the 1000-th digit that Alan writes?

20. _____

21. The big rectangle is divided into 9 rectangles by lines parallel to the sides. If the areas of some of the little rectangles are as shown below, what is the value of x? Write the answer as a common fraction.

21. _____ units^2

3	4	
4		5
	5	x

22. Let r be the number of 8-letter "words" that can be formed using all the letters of the word "RICHMOND". Let v be the number of 9-letter "words" that can be formed using all the letters of the word "VANCOUVER". Express the ratio r/v as a common fraction.
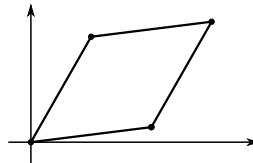
22. _____

23. What is the smallest positive integer whose square is divisible by every integer from 1 to 10?

23. _____

24. Three consecutive vertices of a rhombus have coordinates $(4,7)$, $(0,0)$, and $(8,1)$ respectively. What is the area of the rhombus?
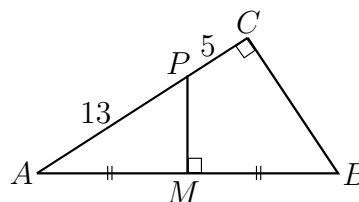
24. _____ units^2

25. What is the smallest prime number which is larger than 293?

25. _____

26. Triangle ABC is right-angled at C, and $AC > BC$. The perpendicular bisector of the hypotenuse AB meets the hypotenuse at M and meets AC at P. Given that $AP = 13$ and $PC = 5$, what is the ratio of the area of $\triangle APM$ to the area of $\triangle ABC$? Express the answer as a common fraction.

26. _____

Bull's-eye, Page 1: Problem Solving

1. Alan says to Beth: "Give me $100, and I shall become twice as rich as you." Beth replies: "Give me $10, and I shall become six times as rich as you." How many dollars does Alan have?

1. _____ dollars

2. A merchant carrying rice passes through three customs posts. At the first post, he has to give up one-third of his rice. At the second post, he has to give up one-fifth of what remains, and at the third post, one-seventh of what remains. He ends up with 5 measures of rice. How many measures of rice did he start out with? Express the answer as a common fraction.

2. _____ measures

3. Painter A can paint a room in 3.3 days, B can paint the room in 5.5 days, and C can paint the room in 6.6 days. They all worked together painting the room for 1 day. Then B and C got fired. How many (additional) days will it take for A to finish the job? Express the answer as a decimal, to the nearest tenth.

3. _____ days

4. The plane was full when it left Vancouver. In Kelowna, half the people got off and 28 got on. In Calgary, half the people got off, 40 got on, and the plane was full again. How many people were on the plane when it left Vancouver?

4. _____ people

Bull's-eye, Page 2: Numbers and Combinatorics

5. How many six-letter "words" can be made using three A's and three B's if no two A's can be next to each other?

5. _____ words

6. The first number in a sequence is 2, and the second number is 3. Each new number in the sequence is obtained by dividing the previous number by the one before that. (Thus the third number is $\frac{3}{2}$.) What is the 100-th number in the sequence? Express the answer as a common fraction.

6. _____

7. What is the reciprocal of the repeating decimal $0.\overline{027}$?

7. _____

8. A 4×4 square is divided into sixteen 1×1 squares as shown. Two different squares are chosen at random from these sixteen squares. What is the probability that the two chosen squares have *exactly one* vertex in common? Express the answer as a common fraction.

8. _____
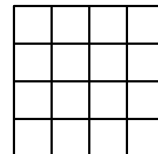
Bull's-eye, Page 3: Geometry

9. Two crystal pyramids each have a 2×2 square base. The two bases are cemented together to make a new crystal. Let V be the number of vertices of the new crystal, E the number of edges, and F the number of faces. What is the value of $V - E + F$?

9. _____

10. The vertices of a triangle are at $(0,0)$, $(48,1)$, and $(50,2)$. How many units^2 are in the area of the triangle?

10. _____ units^2

11. A circle of radius 3 meets a circle of radius 4 at points P and Q. The tangent lines at P to the two circles are perpendicular to each other. What is the length of the line segment PQ? Express the answer as a decimal, to the nearest tenth.

11. _____ units
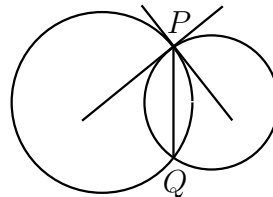
12. Triangle ABC is right-angled, with hypotenuse $AB = \sqrt{1001}$. Point D is on side CA, with $CB = CD$ and $DA = 11$. What is the area of triangle ABC?

12. _____ units^2
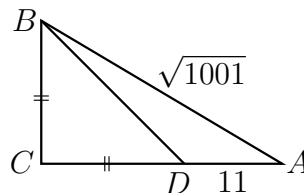
1. How many of the multiples of 10 from 10 to 10^6 (inclusive) are perfect squares?

   1. _____

2. The line with equation $y = 2x + 4$ meets the line with equation $y = -(5 + 6x)$ at the point with coordinates (a, b). What is the value of $a + b$? Express the answer as a common fraction.

   2. _____

3. Two cards are dealt from a well-shuffled standard 52-card deck. What is the probability the cards are both of the same suit (that is, that both are spades, or both are hearts, or both are diamonds, or both are clubs)? Express the answer as a common fraction.

   3. _____

4. What is the smallest possible value of $x + y$, given that x, y, and z are positive integers, z is odd, and $x^2 + y^2 = z^3$?

   4. _____

Co-op, Page 2: Team answers must be on the *coloured* page.
   Answers on a white page will not be graded.

5. A small island country in the middle of the ocean has the shape of   5. _____ km^2
   a triangle with sides 10, 12, and 18 *kilometres.* The country has
   exclusive economic jurisdiction over all parts of the ocean that are
   within 200 *nautical miles* of its shores. (A nautical mile is 1.852
   kilometres.) What is the area, in km^2, of the part of the ocean
   over which the country has exclusive economic jurisdiction? Give the
   answer to the nearest 1000 km^2 (thus your answer should have three
   0's at the end). Assume the earth is flat.

6. A positive integer is called *square-free* if it is not divisible by any   6. _____
   perfect square greater than 1. For example, 1, 2, and 6 are square-
   free, while 4 and 18 are not. Two fair standard dice are tossed. What
   is the probability that the product of the two numbers obtained is
   square-free? Express the answer as a common fraction.

7. Call an integer *lucky* if its decimal representation has two or more   7. _____ integers
   consecutive 8's. For example, 1881 is lucky, as is 8882, but 8087,
   1289, and 4321 are not lucky. How many of the integers in the
   interval from 1000 to 9999 are lucky?

Co-op, Page 3: Team answers must be on the *coloured* page.
Answers on a white page will not be graded.

8. A right circular cone has base radius 3 cm and height 4 cm. What is the number of cm^2 in the total surface area (including the base) of the cone? Express the answer as a decimal, rounded to the nearest tenth of a cm^2. Note that π is approximately equal to 3.14159.

8. _____ cm^2

9. You have a large number of 10 cent coins, 25 cent coins, and 1 dollar coins, and no other coins. In how many ways can you make change for a 10 dollar bill?

9. _____ ways

10. What is the smallest possible value of $\left|\sqrt{2} - a/b\right|$, where a and b are positive integers and $b \leq 20$? Express the answer in scientific notation, correct to 4 significant digits. For example, an answer like 4.567×10^{-3} is of the right shape.

10. _____