# List 2 of Questions from 2006
Provincial Competition

From Bull's Eye Stage

1. Wolf can eat a lamb in 1/4 of an hour. Bear can eat a lamb in 1/5 of an hour. How many hours will it take for Wolf and Bear, dining together at their usual speed, to eat a lamb? Express your answer as a common fraction.

   1. _____ hours

2. There are six balls in an urn, with the numbers 1, 2, 3, 4, 5, and 6 written on them. You reach into the urn and simultaneously remove two randomly chosen balls. What is the probability that the sum of the numbers on these two balls is equal to 6? Express your answer as a common fraction.

   2. _____

3. A small produce store has two cashiers. In how many different ways can 5 customers line up to pay? A sample lineup is given below. Note that all the customers could line up in front of one cashier.

   3. _____ ways

   D   C
   A   B
   E
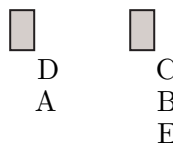
4. The figure below is a regular hexagon. Each of the six sides has length 1 unit. What is the sum of the lengths of all the diagonals? Express your answer in simplest radical form. Note that there are three diagonals through each corner.

   4. _____ units
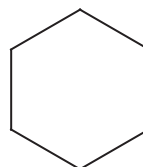
From Blitz Stage

1. What is the smallest prime whose square does not divide 40!?

   1. _____

2. Richie has 3 dimes, 3 quarters, and 3 loonies. How many different non-zero amounts of money can he make by using one or more of these 9 coins?

   2. _____ amounts

3. Sixty percent of sixty percent of a number is 666. What is the number?

   3. _____

4. If the shaded region below has area 150 cm^2, what is the value of x? Give your answer in simplest radical form.

   4. _____ cm

   x 3
   3 | | 3
   x | | x
        x 3
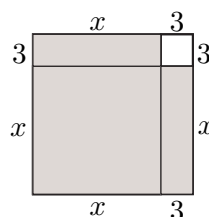
5. What is the smallest positive value of $6 - \dfrac{2006}{n}$, as n ranges over the positive integers? Express your answer as a common fraction.

5. _____

6. A group of 5 students splits into two study groups. (A study group has at least two students.) In how many ways can this be done?

6. _____ ways

7. In how many ways can 100 be expressed as the sum of two (positive) primes? (The decompositions $100 = 3 + 97$ and $100 = 97 + 3$ are considered the same.)

7. _____ ways

8. The first term of a sequence is 1. If x is any term in the sequence, then the next term is the reciprocal of $1 + x$. (So the second term is $1/2$.) What is the product of the first 6 terms of the sequence? Express your answer as a common fraction.

8. _____

From Co-op Stage

1. Calculators are packaged in boxes of 12 or boxes of 25. (Boxes must be full.) What is the least number of boxes required to package 2006 calculators?

1. _____ boxes

2. Two triangles with sides $3\,\text{cm}$, $4\,\text{cm}$, and $5\,\text{cm}$ are placed on top of each other so that the right angles coincide but the triangles do not. What is the number of cm^2 in the region of overlap of the two triangles? Express your answer as a common fraction.

2. _____ cm^2
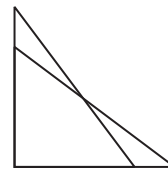
3. Let N be the smallest positive integer such that (i) 24 divides N and (ii) N has exactly 24 positive divisors. What is the value of N? Note that for any positive integer k, both 1 and k are divisors of k.

3. _____

From Face-off Stage

1. The length of a rectangle is four times the width. If the perimeter of the rectangle is 5 metres, how many square metres are in the area of the rectangle?

1. _____

2. When N is divided by 11, the quotient is 12 and the remainder is 7. What is the value of N?

2. _____

3. A cup of flour has 400 calories. A cup of lard has 1700 calories. A pie crust is made using two cups of flour and one cup of lard. How many percent of the calories in the pie crust come from lard?

3. _____

4. Express
$$\frac{1+2+3+4+5+6}{1+2+3+4+5+6+7}$$
as a fraction in lowest terms.

4. _____

5. What is the value of $\dfrac{27.3}{0.13}$?

5. _____

6. What is the sum of the positive integers that divide 24?

6. _____

7. A GMC Yukon on average uses 21.3 litres of gas to travel 100 km. A Toyota Prius uses 4.1 litres of gas to travel 100 km. Beti drives 10,000 km a year. How many dollars in gas costs would she save in a year if she drove a Prius instead of her Yukon? Assume that gas costs \$1.00 per litre.

7. _____

8. What is the largest integer x such that $\dfrac{60}{12-x}$ is an integer?

8. _____

9. The two equal sides of the isosceles triangle below each have length 3 cm. The third side has length 2 cm. What is the area of the triangle, in square cm?

9. _____
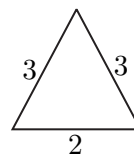
10. The product of 12 positive integers is equal to 12. What is the smallest possible sum of the 12 integers?

10. _____

11. Alice and Bob select, independently and at random, a positive integer that divides 16. What is the probability that they select the same number? Express your answer as a common fraction.

11. _____

12. What common fraction is halfway between $\dfrac{3}{4}$ and $\dfrac{4}{3}$ on the number line?

12. _____

13. If 4 distinct circles are drawn, what is the largest possible number of points that lie on more than one circle?

13. _____

14. What is the sum of the prime factors of $2^8 - 1$?

14. _____

15. Sixty people are arranged in 3 rows. If the back row has three more people than the middle row, and the middle row has three more people than the front row, how many people are in the front row?

15. _____

16. The line segment that joins the points $(-4, 2)$ and $(3, 0)$ is drawn. What is the y-coordinate of the point where this line segment meets the y-axis? Express your answer as a common fraction.

16. _____
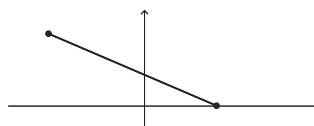
17. A circle has radius 7 units. How many units are in the circumference of the circle? Give the answer rounded to the nearest integer.

17. _____

18. Let $f(x) = x^2 + x + 41$. What is the value of $f(40) - f(-40)$?

18. _____

19. If $4! \times 4! \times N = 8!$, what is the value of N?

19. _____

20. Aleph started with \$2700. He gave one-third of his money to Beth and four-fifths of the rest to Gimel. How many dollars does Aleph have left?

20. _____