4.1 Antiderivatives

Basic Question

What function has derivative $f(x)$?

If $F'(x) = f(x)$, we call $F(x)$ an antiderivative of $f(x)$.

Examples

$\frac{\mathrm{d}}{\mathrm{d}x}[x^2] = 2x$, so x^2 is an antiderivative of $2x$.

$\frac{\mathrm{d}}{\mathrm{d}x}[x^2 + 5] = 2x$, so $x^2 + 5$ is (also) an antiderivative of $2x$.

What is the most general antiderivative of $2x$?

Antiderivatives

Find the most general antiderivative for the following equations.

$$f(x) = 17$$

$$f(x) = m$$

where m is a constant.

differentiation fact		antidifferentiation fact
$\frac{d}{dx}[x^2] = 2x$	$\Longrightarrow$	antideriv of $2x$:
$\frac{d}{dx}[x^3] = 3x^2$	$\Longrightarrow$	
$\frac{d}{dx}[x^4] = 4x^3$	$\Longrightarrow$	
$\frac{d}{dx}[x^5] = 5x^4$	$\Longrightarrow$	

antideriv of x^n:

Power Rule for Antidifferentiation

The most general antiderivative of x^n is $\dfrac{1}{n+1}x^{n+1} + c$ if $n \neq -1$

- $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad\qquad \Big] = x^5$
- $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad\qquad \Big] = x^3$
- $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad\qquad \Big] = \dfrac{1}{2}x^3$

Power Rule for Antidifferentiation

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- $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad\qquad\Big] = 5x^2 - 15x + 3$
- $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big[\qquad\qquad\qquad\Big] = 13\left(5x^{14} - 3x^{3/7} + 52e^x\right)$

Find the most general antiderivatives.

$$f(x) = \cos x$$

$$f(x) = \sin x$$

$$f(x) = \sec^2 x$$

$$f(x) = \frac{1}{1+x^2}$$

$$f(x) = \frac{1}{1+x^2+2x}$$

Find the most general antiderivatives.

$$f(x) = 17\cos x + x^5$$

$$f(x) = \frac{23}{5 + 5x^2}$$

$$f(x) = \frac{23}{5 + 125x^2}$$

Find the most general antiderivatives.

$$f(x) = \frac{1}{x},\ x > 0$$

$$f(x) = 5x^2 - 32x^5 - 17$$

$$f(x) = \csc x \cot x$$

$$f(x) = \frac{5}{\sqrt{1 - x^2}} + 17$$

Chose Your Own Adventure

Antiderivative of $\sin x \cos x$:

A. $\cos x \sin x + c$

B. $-\cos x \sin x + c$

C. $\sin^2 x + c$

D. $\frac{1}{2}\sin^2 x + c$

E. $\frac{1}{2}\cos^2 x \sin^2 x + c$

In general, antiderivatives of x^n have the form $\frac{1}{n+1}x^{n+1}$. What is the single exception?

A. $n = -1$

B. $n = 0$

C. $n = 1$

D. $n = e$

E. $n = 1/2$

All the Adventures are Calculus, Though

Suppose the velocity of a particle at time t is given by $v(t) = t^2 + \cos t + 3$. What function gives its position?

A. $s(t) = 2t - \sin t$
B. $s(t) = 2t - \sin t + c$
C. $s(t) = t^3 + \sin t + 3t + c$
D. $s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$
E. $s(t) = \frac{1}{3}t^2 - \sin t + 3t + c$

Suppose the velocity of a particle at time t is given by $v(t) = t^2 + \cos t + 3$, and its position at time 0 is given by $s(0) = 5$. What function gives its position?

A. $s(t) = \frac{1}{3}t^3 + \sin t + 3t$
B. $s(t) = \frac{1}{3}t^3 + \sin t + 3t + 5$
C. $s(t) = \frac{1}{3}t^3 + \sin t + 3t + c$
D. $s(t) = 5t + c$
E. $s(t) = 5t + 5$

Find all functions $f(x)$ with $f(1) = 5$ and $f'(x) = e^{3x+5}$.

Let $Q(t)$ be the amount of a radioactive isotope in a sample. Suppose the sample is losing $50e^{-5t}$ mg per second to decay. If $Q(1) = 10e^{-5}$mg, find the equation for the amount of the isotope at time t.

Suppose $f'(t) = 2t + 7$. What is $f(10) - f(3)$?