PUTNAM PRACTICE SET 7

PROF. DRAGOS GHIOCA

Problem 1. Let $P \in \mathbb{C}[x]$ be a polynomial of degree $n \geq 1$ with the property that $P(k) = \frac{1}{\binom{n+1}{k}}$ for each $k = 0, 1, \ldots, n$. Find $P(n+1)$.

Problem 2. Find the maximum value for $m^2 + n^2$ where $1 \leq m, n \leq 2019$ and moreover $(n^2 - mn - m^2)^2 = 1$.

Problem 3. We define the recurrence sequence $\{a_n\}_{n \geq 1}$ given by:

$$a_1 = 1 \text{ and } a_{n+1} = \frac{1 + 4a_n + \sqrt{1 + 24a_n}}{16} \text{ for each } n \geq 1.$$

Find a_{2019}.

Problem 4. Let $1 \leq r \leq n$ be integers. We consider the set $\mathcal{M}$ the set of all subsets of $\{1, 2, \ldots, n\}$ consisting of exactly r elements. For each $S \in \mathcal{M}$, we let m_S be the smallest element contained in S. Find the arithmetic mean of all m_S (for $S \in \mathcal{M}$).