The University of British Columbia

# Mathematics 335 (201)

# Final Examination

23 April 2010

Time: 150 minutes

Full Name: ______________________ Student # : ______________

Signature: ______________________________

This Examination paper consists of 9 pages (including this one). Make sure you have all 9.

Instructions:

One 8.5" x 11" sheet plus one acetate allowed. One calculator allowed. No communication devices allowed.

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Rules:

- Each candidate must be prepared to produce, upon request, a UBC card for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
  - having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
  - speaking or communicating with other candidates; and
  - purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

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marking:

| **Q1** | | **Q2** | | **Q3** | | **Q4** | | **Q5** | | **Q6** | | **Q7** | |
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| | | | | | | | | | | | | TOTAL | |

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Name of Instructor: Mark Mac Lean

**Q1** [10 marks]

Short answers: give one or two sentences about the following. You are encouraged to use examples and figures.

($a$) symmetry of a geometric figure

($b$) platonic solid

($c$) rational number

($d$) great circle on a sphere

($e$) dual of a solid

**Q2** [10 marks]

Recall from class that we observed three relationships between vertices, edges, and faces in the platonic solids:

$$\begin{aligned} pF &= 2E && (1) \\ qV &= 2E && (2) \\ V - E + F &= 2 && (3) \end{aligned}$$

where $p$ is the number of edges on each face, and $q$ is the number of edges (or faces) meeting at a vertex.

($a$) Solve (1) and (2) for $F$ and $V$ in terms of $E$.

($b$) Substitute the results into (3) to get an expression entirely in terms of $E$. As we did in class, manipulate this to get

$$\frac{1}{q} + \frac{1}{p} = \frac{1}{2} + \frac{1}{E}.$$

($c$) Explain why the above equality can be used to say that $p$ and $q$ must satisfy

$$\frac{1}{q}+\frac{1}{p}>\frac{1}{2}$$

regardless of the number of edges.

($d$) Find all the possible pairs of values $(p, q)$ that satisfy this inequality. Why is $p \geq 3$? Identify the solid associated to each pair $(p, q)$.

($e$) How does this tell you there are only 5 platonic solids?

**Q3** [10 marks]

(*a*) Consider a triangle on the surface of a sphere that has two angles that each have measure $45°$. Is it possible for the third angle of this triangle to be exactly $90°$? Why or why not? It may help for you to draw diagrams.

(*b*) Suppose that you draw a triangle on the surface of a cone such that the interior of the triangle does *not* contain the vertex of the cone. What is the sum of the angles of this triangle? Why? What happens if you include the vertex of the cone in the interior of the triangle? It may help for you to draw diagrams.

**Q4** [10 marks]

Consider the rotational symmetries of an equilateral triangle. These are the identity $I$, rotation by 120 degrees clockwise $R_{120}$, and rotation by 240 degrees clockwise $R_{240}$. The rotational symmetries form a group of symmetries which is a subgroup of the whole group of symmetries (which includes the reflections) of the equilateral triangle. As we did in class, let us use $\circ$ to designate composing two symmetries and also use the convention that when we write $a \circ b$, we mean that $a$ is done first and $b$ second.

$(a)$ Write down the group table for the rotational symmetries of the triangle.

$(b)$ Solve $R_{120} \circ x = I$ for $x$.

$(c)$ Solve $R_{120} \circ x = R_{240}$ for $x$.

$(d)$ Can you solve $x \circ x = I$? Explain your answer.

**Q5** [10 marks]

($a$) What is an *irrational number*? Give two examples other than $\sqrt{3}$.

($b$) Show that $\sqrt{3}$ is irrational. (Hint: Assume $\sqrt{3}$ is a rational number and derive a contradiction to this assumption.)

**Q6** [10 marks]

($a$) Show there is no smallest positive rational number. You may draw a diagram to help in your explanation.

($b$) Suppose you have the two rational numbers $\frac{1}{2}$ and $\frac{2}{3}$. Show that you can construct infinitely many rational numbers between these two given ones. You may draw a diagram to help in your explanation.

**Q7** [10 marks]

Consider your past and present experiences learning mathematics. As a student taking Math 335 this term, what did you learn about learning mathematics?