THE UNIVERSITY OF BRITISH COLUMBIA

Math 335 Section 201

No calculators allowed
One two-sided information sheet allowed
Final exam begins at 12 noon and ends at 2:30 pm

FINAL EXAM April 24, 2009

NAME

STUDENT NUMBER

[10]] 1. Cantor proved that for any set S, the set P(S) of subsets of S has a larger cardinality than S.

Let S = {"why", "is", "this", "not", "the", "wrong", "right", "question", "answer"} be a set of words. Suppose you associate each element of S with a subset of S as follows:

"why" is associated with {"why", "this"}
"is" is associated with {"why", "this", "not"}
"this" is associated with {"is"}
"not" is associated with {"why", "is", "this", "not", "the"}
"the" is associated with { }
"wrong" is associated with {"this", "is", "wrong", "right"}
"right" is associated with {"wrong", "is", "this"}
"question" is associated with {"is", "this", "question", "wrong"}
"answer" is associated with {"why", "this", "question", "right"}

There are many subsets of S not associated with any element of S; however, what subset would you find using the construction that occurs in the proof of Cantor's theorem?

2.

[5] (a) April 24, 2009, is a Friday. What day of the week will April 24, 2010 be? Answer the same question for April 24, 2012.

[5] (b) You started a long mathematics exam at noon. You were told that you could work as long as you liked. You worked 487 hours straight. At what time of day did you finish?

[10] 3. Prove that $\sqrt{5}$ is irrational.

[10] 4. A 1-dimensional cube is a line segment; a 2-dimensional cube is a square; a 3-dimensional cube is a cube ; and so on. Complete the following table:

Dimension of Cube	# of vertices	# of edges	# of 2D faces	# of 3D "faces"
1				
2				
3				
4				

[10] 5. A voter with a rowboat finds herself in an unusual situation. On one side of a river is Bob Dole, Bill Clinton, and a big bag of Burger King Whoppers. The voter must get Dole, Clinton, and the bag of burgers across the river to the other side. Her boat, however, is large enough for her and just one other item or person. If she leaves Dole alone with Clinton, then Dole (an old war veteran) will beat up Clinton. If she leaves Clinton alone with the bag of Whoppers, then Clinton (a big burger lover) will devour the entire contents of the bag. Of course, Dole, Clinton, and the bag are all incapable of rowing the boat across the river. Is it possible for this voter to get all her cargo across the river? If so, carefully explain her method; if not, carefully explain why not.

6. Here is a chart of gas mileage (MPG) for various cars.

Make and Model	**MPG (highway driving)**
Honda Insight	**66**
VW Passat	**24**
Chevy Malibu	**30**
Pontiac GTO	**21**
Toyota Celica	**36**
Land Rover Range Rover	**16**

[5] Compute the mean and median MPG.

[5] Compute the standard deviation.

7. Suppose the course offerings at the University of Texas at Austin are simplified. Only 20 classes are offered, each meets at a different hour each week in the stadium, and there is no enrollment limit on any course. Each student is required to take exactly four courses. There are no prerequisites, and every student can take any class. UT at Austin has about 48,000 students.

[5] (a) If the students choose independently, how many different course schedules are possible?

[5] (b) Will some students have to have the exact same program or not? Justify your answer.

8.
[5] Solve each equation for x

(a) $\frac{x}{1} = \frac{1}{x-1}$ (b) $\frac{x}{3} = \frac{2}{x-4}$

[5] Take a Golden Rectangle and attach a square to the longer side so that you create a new larger rectangle. Is this new rectangle a Golden Rectangle? If so, prove it. If not, show why not.

9.

[5] (a) Express each of the following numbers as a product of primes.

(i) 1863

(ii) 2187

(iii) 3003

[5] (b) Compute the greatest common divisor and the least common multiple of each of the following pairs of integers

(i) (55, 66)

(ii) (91, 14)

(iii) (76, 57)

10.

[5] (a) Determine the integer nearest to the given fraction

(i) $\frac{75}{16}$

(ii) $\frac{75}{17}$

(iii) $\frac{750}{200}$

[5] (b) Evaluate

(i) $\left(\frac{2}{3}\right)^{10} \div \left(\frac{2}{3}\right)^{8}$

(ii) $\frac{3^5}{2^5} \div \frac{3^6}{2^6}$

(iii) $\left[\left(\frac{2}{5}\right)^{4} \times \left(\frac{2}{5}\right)^{3}\right]^{2} \div \left(\frac{2}{5}\right)^{12}$