The University of British Columbia

Final Examination - December 16, 2010

Mathematics 317

Instructor: Katherine Stange

Closed book examination

Time: 3 hours

Name _____

Signature _____

Student Number _____

Special Instructions:

- Be sure that this examination has 17 pages. Write your name on top of each page. Do not detach any pages.
- No calculators or notes are permitted.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations • Each candidate must be prepared to produce, upon request, a UBCcard for identification. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. • Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action. (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners. (b) Speaking or communicating with other candidates. (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received. • Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

• Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1	7
2	9
3	8
4	8
5	14
6	8
7	7
8	7
9	22
10	10
Total	100

Problem 1 of 10 [7 points]

Let $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, 4t \rangle$ be the position vector of a particle as a function of time $t \ge 0$.

(a) (2 points) Find the velocity of the particle as a function of time t.

(b) (5 points) Find the arclength of its path between t = 1 and t = 2.

December 2010 Math 317 Name:

Problem 2 of 10 [9 points]

Let C be the upper half of the unit circle centred on (1,0) (i.e. that part of the circle which lies above the x axis), oriented clockwise. Compute the line integral

 $\int_C xy \ dy.$

Problem 3 of 10 [8 points]

Let S be the surface given by

$$\mathbf{s}(u,v) = \langle u+v, u^2+v^2, u-v \rangle, \quad -2 \le u \le 2, -2 \le v \le 2$$

(a) (4 points) Find the tangent plane to the surface at the point (2, 2, 0).

(b) (2 points) This is a surface you are familiar with. What surface is it (it may be just a portion of one of the following)? Circle one:

sphere helicoid ellipsoid saddle parabolic bowl cylinder cone plane

(c) (2 points) In which direction does the parametrisation orient the surface? Circle the correct choices: In the (positive / negative) (x / y / z) direction.

December 2010 Math 317 Nam

Name:

Problem 4 of 10 [8 points]

Let

$$\mathbf{F}(x,y) = \langle y^2 - e^{-y^2} + \sin x, \ 2xye^{-y^2} + x \rangle.$$

Let C be the boundary of the triangle with vertices (0,0), (1,0) and (1,2), oriented counterclockwise. Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

Hint: Don't do the integral directly.

December 2010 Math 317 Name:

Problem 5 of 10 [14 points]

Let

$$\mathbf{F}(x,y,z) = \langle \frac{y}{x} + x^{1+x^2}, \ x^2 - y^{1+y^2}, \ \cos^5(\ln z) \rangle.$$

(a) (2 points) Write down the domain D of \mathbf{F} .

- (b) (1 points) Circle the correct statement(s):
 - (a) D is connected.
 - (b) D is simply connected.
 - (c) D is disconnected.
- (c) (2 points) Compute $\nabla \times \mathbf{F}$.

December 2010 Math 317 Name: _____

(d) (8 points) Let C be the square with corners $(3 \pm 1, 3 \pm 1)$ in the plane z = 2, oriented clockwise (viewed from above, i.e. down z-axis). Compute

$$\int_{C} \mathbf{F} \cdot d\mathbf{r}.$$

Hint: Don't do it directly.

Let

$$\mathbf{F}(x, y, z) = \langle e^{-y^2} + y^{1+x^2} + \cos(z), \ -z, \ y \rangle.$$

Let S be the surface which consists of two parts:

- the portion of the paraboloid $y^2 + z^2 = 4(x+1)$ satisfying $0 \le x \le 3$ and
- the portion of the sphere $x^2 + y^2 + z^2 = 4$ satisfying x < 0.

and is oriented outward. Compute

$$\iint_{S} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

Hint: Don't do it directly.

December 2010 Math 317 Name:

Problem 7 of 10 [7 points]

Let

$$\mathbf{F}(x, y, z) = \langle 1 + z^{1+z^{1+z}}, 1 + z^{1+z^{1+z}}, 1 \rangle.$$

Let S be the portion of the surface

$$x^2 + y^2 = 1 - z^4$$

which is above the xy-plane. What is the flux of **F** downward through S?

December 2010 Math 317 Nam

Name: _____

Page 10 of 17

Problem 8 of 10 [7 points]

Let

$$\mathbf{F}(x,y) = \langle 1, \ y \ g(y) \rangle,$$

and suppose that g(y) is a function defined everywhere with everywhere continuous partials. Show that for any curve C whose endpoints P and Q lie on the x-axis,

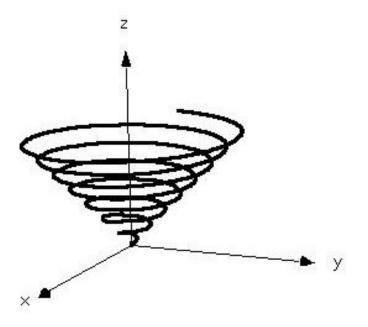
distance between
$$P$$
 and $Q = \left| \int_{C} \mathbf{F} \cdot d\mathbf{r} \right|$.

Problem 9 of 10 [22 points]

Short Answer. Only your answer counts.

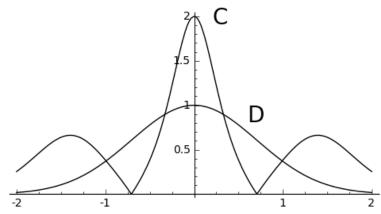
(a) (1 point) In the curve shown below (a helix lying in the surface of a cone), is the curvature increasing, decreasing, or constant as z increases?

Circle the answer: increasing decreasing constant



(b) (2 points) Of the two functions shown below, one is a function f(x) and one is its curvature $\kappa(x)$. Which is which?

f(x) is (circle one): $C \quad D$



(c) (3 points) Let C be the curve of intersection of the cylinder $x^2 + z^2 = 1$ and the saddle xz = y. Parametrise C.

$$\mathbf{r}(t) =$$
 domain:

(d) (4 points) Let H be the helical ramp (also known as a helicoid) which revolves around the z-axis in a clockwise direction viewed from above, beginning at the y-axis when z = 0, and rising 2π units each time it makes a full revolution. Let S be the the portion of H which lies outside the cylinder $x^2 + y^2 = 4$, above the z = 0 plane and below the z = 5 plane.

Choose one of the following functions and give the domain on which the function you have chosen parametrises S. (Hint: Only one of the following functions is possible.)

- (a) $s(u, v) = \langle u \cos v, u \sin v, u \rangle$ (b) $s(u, v) = \langle u \cos v, u \sin v, v \rangle$
- (c) $s(u, v) = \langle u \sin v, u \cos v, u \rangle$ (d) $s(u, v) = \langle u \sin v, u \cos v, v \rangle$

domain:

(e) (2 points) Write down a parametrised curve of zero curvature and arclength 1.

$\mathbf{r}(t) =$ domain:

(f) (2 points) If $\nabla \cdot \mathbf{F}$ is a constant C on all of \mathbb{R}^3 , and S is a cube of unit volume such that the flux outward through each side of S is 1, what is C?

C =

(g) (2 points) Let

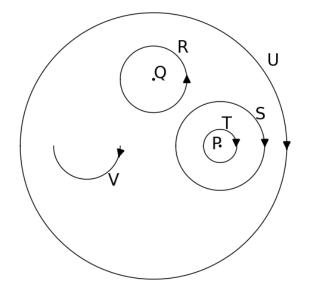
$$\mathbf{F}(x,y) = \langle ax + by, cx + dy \rangle.$$

Give the full set of a, b, c and d such that **F** is conservative.

(h) (1 point) If $\mathbf{r}(s)$ has been parametrised by arclength (i.e. s is arclength), what is the arclength of $\mathbf{r}(s)$ between s = 3 and s = 5?

arclength:

- (i) (2 points) Let \mathbf{F} be a 2D vector field which is defined everywhere *except* at the points marked P and Q. Suppose that $\nabla \times \mathbf{F} = 0$ everywhere on the domain of \mathbf{F} . Consider the five curves R, S, T, U, and V shown in the picture. Which of the following is *necessarily* true?
 - (a) $\int_{S} \mathbf{F} \cdot d\mathbf{r} = \int_{T} \mathbf{F} \cdot d\mathbf{r}$
 - (b) $\int_{B} \mathbf{F} \cdot d\mathbf{r} = \int_{S} \mathbf{F} \cdot d\mathbf{r} = \int_{T} \mathbf{F} \cdot d\mathbf{r} = \int_{U} \mathbf{F} \cdot d\mathbf{r} = 0$
 - (c) $\int_R \mathbf{F} \cdot d\mathbf{r} + \int_S \mathbf{F} \cdot d\mathbf{r} + \int_T \mathbf{F} \cdot d\mathbf{r} = \int_U \mathbf{F} \cdot d\mathbf{r}$
 - (d) $\int_U \mathbf{F} \cdot d\mathbf{r} = \int_R \mathbf{F} \cdot d\mathbf{r} + \int_S \mathbf{F} \cdot d\mathbf{r}$
 - (e) $\int_V \mathbf{F} \cdot d\mathbf{r} = 0$

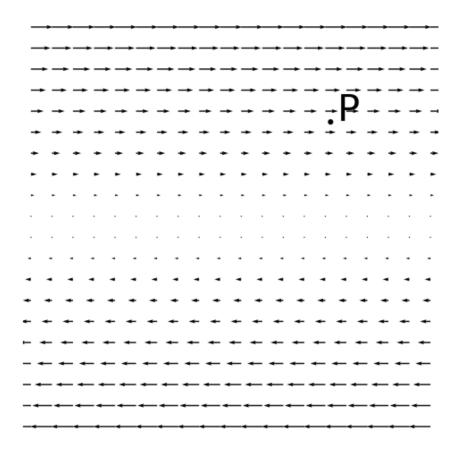


(j) (2 points) Write down a 3D vector field \mathbf{F} such that for all closed surfaces S, the volume enclosed by S is equal to

$$\iint_{S} \mathbf{F} \cdot d\mathbf{S}.$$

$$\mathbf{F}(x,y,z) =$$

(k) (1 point) Consider the vector field \mathbf{F} shown below. curl \mathbf{F} at P is (circle one): positive negative zero



Problem 10 of 10 [10 points]

Say whether the following statements are true (\mathbf{T}) or false (\mathbf{F}) . Only your answer counts. You will get 1 point for each correct answer, 0 for each wrong or unanswered.

- **T F** If **F** is a 3D vector field defined on all of \mathbb{R}^3 , and S_1 and S_2 are two surfaces with the same boundary, but $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} \neq \iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$, then div **F** is not zero anywhere.
- **T F** If **F** is a vector field satisfying curl $\mathbf{F} = 0$ whose domain is not simply-connected, then **F** is not conservative.
- **T F** The osculating circle of a curve C at a point has the same unit tangent vector, unit normal vector, and curvature as C at that point.
- $\mathbf{T} \ \mathbf{F}$ A planet orbiting a sun has period proportional to the cube of the major axis of the orbit.
- **T F** For any 3D vector field **F**, $\operatorname{div}(\operatorname{curl} \mathbf{F}) = 0$.
- **T F** A field whose divergence is zero everywhere in its domain has $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$ for all closed surfaces S in its domain.
- **T F** The gravitational force field is conservative.
- **T F** If **F** is a field defined on all of \mathbf{R}^3 such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 3$ for some curve *C*, then $\nabla \times \mathbf{F}$ is non-zero at some point.
- **T F** The normal component of acceleration for a curve of constant curvature is constant.
- $\mathbf{T} \mathbf{F}$ The curve defined by

$$\mathbf{r}_1(t) = \cos(t^4) \,\mathbf{i} + 3t^4 \,\mathbf{j}, \quad -\infty < t < \infty,$$

is the same as the curve defined by

$$\mathbf{r}_2(t) = \cos t \, \mathbf{i} + 3t \, \mathbf{k}, \quad -\infty < t < \infty.$$

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