The University of British Columbia
Final Examination - April 17, 2009

Mathematics 317 Section 201
Instructors: Mahta Khosravi and Hendryk Pfeiffer

Closed book examination. Time: 2.5 hours

Name ____________________ **Signature** ____________________

Student Number ________________

Special Instructions:

- Be sure that this examination has 15 pages. Write your name on top of each page.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

1		10
2		10
3		14
4		10
5		10
6		14
7		12
8		12
9		8
Total		100

Problem 1 of 9 [10 points]

Assume the paraboloid $z = x^2 + y^2$ and the plane $2x + z = 8$ intersect in a curve C. C is traversed counter-clockwise if viewed from the positive z-axis.

(1) [4 points] Parameterize the curve C.

(2) [6 points] Find the unit tangent vector $\mathbf{T}$, the principal normal vector $\mathbf{N}$, the binormal vector $\mathbf{B}$ and the curvature κ all at the point $\langle 2, 0, 4\rangle$.

Problem 2 of 9 [10 points]

A particle of mass $m = 1$ has position $\mathbf{r}_0 = \mathbf{j}$ and velocity $\mathbf{v}_0 = \mathbf{i} + \mathbf{k}$ at time $t = 0$. The particle moves under a force

$$\mathbf{F}(t) = \mathbf{j} - \sin t\, \mathbf{k},$$

where t denotes time.

(1) [5 points] Find the position $\mathbf{r}(t)$ of the particle as a function of t.

(2) [2 points] Find the position $\mathbf{r}_1$ of the particle when it crosses the plane $x = \pi/2$ for the first time after time $t = 0$.

(3) [3 points] Determine the work done by $\mathbf{F}$ in moving the particle from $\mathbf{r}_0$ to $\mathbf{r}_1$.

Problem 3 of 9 [14 points]

Consider the vector field $\mathbf{F}$ defined as

$$\mathbf{F}(x,y,z) = \left\langle (1+ax^2)ye^{3x^2} - bxz\cos(x^2z), xe^{3x^2}, x^2\cos(x^2z) \right\rangle,$$

where a and b are real valued constants.

(1) [4 points] Compute curl $\mathbf{F}$.

(2) [2 points] Determine for which values a and b the vector field $\mathbf{F}$ is conservative.

(3) [5 points] For the values of a and b obtained in part (2), find a potential function f such that $\nabla f = \mathbf{F}$.

(4) [3 points] Evaluate the line integral

$$\int_C \left(ye^{3x^2} + 2xz\cos(x^2z) \right) dx + xe^{3x^2}\,dy + x^2\cos(x^2z)\,dz,$$

where C is the arc of the curve $\langle t, t, t^3 \rangle$ starting at the point $\langle 0,0,0 \rangle$ and ending at the point $\langle 1,1,1 \rangle$. Hint: Notice the difference between this vector field and the conservative vector field.

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Problem 4 of 9 [10 points]

Suppose the curve C is the boundary of the region enclosed between the curves $y = x^2 - 4x + 3$ and $y = 3 - x^2 + 2x$. Determine the value of the line integral

$$\int_C (2xe^y + \sqrt{2 + x^2})\, dx + x^2(2 + e^y)\, dy,$$

where C is traversed counter-clockwise.

Problem 5 of 9 [10 points]

Suppose S is the part of the hyperboloid $x^2 + y^2 - 2z^2 = 1$ that lies inside the cylinder $x^2 + y^2 = 9$ and above the plane $z = 1$ (*i.e.* for which $z \geq 1$).
Which of the following are parameterizations of S? Write your answer 'yes' (Y) or 'no' (N) in the following box. No explanation required. [2 points for a correct answer, 1 point if you do not answer, 0 if wrong]

	1	2	3	4	5
Y/N					

(1) The vector function

$$\mathbf{r}(u,v) = u\,\mathbf{i} + v\,\mathbf{j} + \frac{\sqrt{u^2+v^2-1}}{\sqrt{2}}\,\mathbf{k}$$

with domain $D = \{\,(u,v) \mid 2 \leq u^2 + v^2 \leq 9\,\}$.

(2) The vector function

$$\mathbf{r}(u,v) = u\sin v\,\mathbf{i} - u\cos v\,\mathbf{j} + \sqrt{\frac{u^2}{2} - \frac{1}{2}}\,\mathbf{k}$$

with domain $D = \{\,(u,v) \mid \sqrt{3} \leq u \leq 3,\ 0 \leq v \leq 2\pi\,\}$.

(3) The vector function

$$\mathbf{r}(u,v) = \sqrt{1+2v^2}\cos u\,\mathbf{i} + \sqrt{1+2v^2}\sin u\,\mathbf{j} + v\,\mathbf{k}$$

with domain $D = \{\,(u,v) \mid 0 \leq u \leq 2\pi,\ 1 \leq v \leq 2\,\}$.

(4) The vector function

$$\mathbf{r}(u,v) = \sqrt{1+u}\sin v\,\mathbf{i} + \sqrt{1+u}\cos v\,\mathbf{j} + \sqrt{\frac{u}{2}}\,\mathbf{k}$$

with domain $D = \{\,(u,v) \mid 2 \leq u \leq 8,\ 0 \leq v \leq 2\pi\,\}$.

(5) The vector function

$$\mathbf{r}(u,v) = \sqrt{u}\cos v\,\mathbf{i} - \sqrt{u}\sin v\,\mathbf{j} + \frac{\sqrt{u+1}}{\sqrt{2}}\,\mathbf{k}$$

with domain $D = \{\,(u,v) \mid 3 \leq u \leq 9,\ 0 \leq v \leq 2\pi\,\}$.

Problem 6 of 9 [14 points] Consider the ellipsoid S given by

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1,$$

with the unit normal pointing outward.

(1) [4 points] Parameterize S. Hint: Use polar coordinates in a suitable way. Do not forget to specify the range of the parameters.

(2) [6 points] Compute the flux $\iint_S \mathbf{F} \cdot d\mathbf{S}$ of the vector field

$$\mathbf{F}(x, y, z) = \langle x, y, z \rangle .$$

(3) [4 points] Verify your answer in (2) using the divergence theorem.

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Problem 7 of 9 [12 points]

Evaluate the line integral

$$\int_C \left(z + \frac{1}{1+z}\right) dx + xz\, dy + \left(3xy - \frac{x}{(z+1)^2}\right) dz,$$

where C is the curve parameterized by

$$\mathbf{r}(t) = \left\langle \cos t, \sin t, 1 - \cos^2 t \sin t \right\rangle, \qquad 0 \le t \le 2\pi.$$

Hint: The curve C bounds the surface $z = 1 - x^2 y,\ x^2 + y^2 \le 1$.

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Problem 8 of 9 [12 points]

Evaluate the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = \left\langle x^3 + \cos(y^2), y^3 + ze^x, z^2 + \arctan(xy) \right\rangle,$$

and S is the surface of the solid region bounded by the cylinder $x^2 + y^2 = 2$ and the planes $z = 0$ and $z = 2x + 3$. The surface is positively oriented (its unit normal points outward).

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Problem 9 of 9 [8 points]

Which of the following statements are true (T) and which are false (F)? Write your answers in the following box. You do not need to give reasons. All real valued functions $f(x, y, z)$ and all vector fields $\mathbf{F}(x, y, z)$ have domain $\mathbb{R}^3$ unless specified otherwise. [1 point for a correct answer, 0.5 points if you do not answer, 0 if wrong]

	1	2	3	4	5	6	7	8
T/F								

(1) If f is a continuous real valued function and S a smooth oriented surface, then

$$\iint_S f\,dS = -\iint_{-S} f\,dS,$$

where '$-S$' denotes the surface S but with the opposite orientation.

(2) Suppose the components of the vector field $\mathbf{F}$ have continuous partial derivatives. If $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = 0$ for every closed smooth surface, then $\mathbf{F}$ is conservative.

(3) Suppose S is a smooth surface bounded by a smooth simple closed curve C. The orientation of C is determined by that of S as in Stokes' theorem. Suppose the real valued function f has continuous partial derivatives. Then

$$\int_C f\,dx = \iint_S \left(\frac{\partial f}{\partial z}\,\mathbf{j} - \frac{\partial f}{\partial y}\,\mathbf{k}\right)\cdot d\mathbf{S}.$$

(4) Suppose the real valued function $f(x, y, z)$ has continuous second order partial derivatives. Then

$$(\nabla f)\times(\nabla f) = \nabla\times(\nabla f).$$

(5) The curve parameterized by

$$\mathbf{r}(t) = \left\langle 2 + 4t^3, -t^3, 1 - 2t^3\right\rangle, \qquad -\infty < t < \infty,$$

has curvature $\kappa(t) = 0$ for all t.

(6) If a smooth curve is parameterized by $\mathbf{r}(s)$ where s is arc length, then its tangent vector satisfies

$$|\mathbf{r}'(s)| = 1.$$

(7) If S is the sphere $x^2 + y^2 + z^2 = 1$ and $\mathbf{F}$ is a constant vector field, then $\iint_S \mathbf{F}\cdot d\mathbf{S} = 0$.

(8) There exists a vector field $\mathbf{F}$ whose components have continuous second order partial derivatives such that $\operatorname{curl}\mathbf{F} = \langle x, y, z\rangle$.

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