

The University of British Columbia
Final Examination - April 19, 2007

Mathematics 317

Instructors: Jim Bryan and Hendryk Pfeiffer

Closed book examination

Time: 3 hours

Name _____ Signature _____

Student Number _____

Special Instructions:

- Be sure that this examination has 14 pages. Write your name on top of each page.
- No calculators or notes are permitted.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBC-card for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Problem 1 of 10 [10 points]

Suppose the curve C is the intersection of the cylinder $x^2 + y^2 = 1$ with the plane $x + y + z = 1$.

- (1.) [4 points] Find a parameterization of C .
- (2.) [3 points] Determine the curvature of C .
- (3.) [3 points] Find the points at which the curvature is maximum and determine the value of the curvature at these points.

Problem 2 of 10 [10 points]

Consider the curve

$$\mathbf{r}(t) = \frac{1}{3} \cos^3 t \mathbf{i} + \frac{1}{3} \sin^3 t \mathbf{j} + \sin^3 t \mathbf{k}.$$

- (1.) Compute the arc length of the curve from $t = 0$ to $t = \frac{\pi}{2}$.
- (2.) Compute the arc length of the curve from $t = 0$ to $t = \pi$.

Problem 3 of 10 [10 points]

Consider the vector field

$$\mathbf{F}(x, y, z) = -2y \cos x \sin x \mathbf{i} + (\cos^2 x + (1 + yz) e^{yz}) \mathbf{j} + y^2 e^{yz} \mathbf{k}.$$

- (1.) [5 points] Find a real valued function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.
- (2.) [5 points] Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the arc of the curve $\mathbf{r}(t) = \langle t, e^t, t^2 - \pi^2 \rangle$, $0 \leq t \leq \pi$, traversed from $(0, 1, -\pi^2)$ to $(\pi, e^\pi, 0)$.

Problem 4 of 10 [10 points]

Evaluate the surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F}(x, y, z) = \langle \cos z + xy^2, x e^{-z}, \sin y + x^2 z \rangle$ and S is the boundary of the solid region enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

Problem 5 of 10 [10 points]

Evaluate the surface integral

$$\iint_S xy^2 dS$$

where S is the part of the sphere $x^2 + y^2 + z^2 = 2$ for which $x \geq \sqrt{y^2 + z^2}$.

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Problem 6 of 10 [10 points]

Evaluate the line integral

$$\int_C (x^2 + y e^x) dx + (x \cos y + e^x) dy$$

where C is the arc of the curve $x = \cos y$ for $-\pi/2 \leq y \leq \pi/2$, traversed in the direction of increasing y .

Problem 7 of 10 [10 points]

Consider the vector field

$$\mathbf{F}(x, y, z) = \frac{x - 2y}{x^2 + y^2} \mathbf{i} + \frac{2x + y}{x^2 + y^2} \mathbf{j} + z \mathbf{k}.$$

- (1.) [2 points] Determine the domain of \mathbf{F} .
- (2.) [3 points] Compute $\nabla \times \mathbf{F}$. Simplify the result.
- (3.) [3 points] Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the circle of radius 2 in the plane $z = 3$, centered at $(0, 0, 3)$ and traversed counter-clockwise if viewed from the positive z -axis, i.e. viewed “from above”.

- (4.) [2 points] Is \mathbf{F} conservative?

Problem 8 of 10 [10 points]

Suppose the curve C is the intersection of the cylinder $x^2 + y^2 = 1$ with the surface $z = xy^2$, traversed clockwise if viewed from the positive z -axis, *i.e.* viewed “from above”. Evaluate the line integral

$$\int_C (z + \sin z) dx + (x^3 - x^2y) dy + (x \cos z - y) dz.$$

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Problem 9 of 10 [10 points]

A physicist studies a vector field $\mathbf{F}(x, y, z)$. From experiments, it is known that \mathbf{F} is of the form

$$\mathbf{F}(x, y, z) = xz \mathbf{i} + (axe^y z + byz) \mathbf{j} + (y^2 - xe^y z^2) \mathbf{k}$$

for some real numbers a and b . It is further known that $\mathbf{F} = \nabla \times \mathbf{G}$ for some differentiable vector field \mathbf{G} .

- (1.) [4 points] Determine a and b .
- (2.) [6 points] Evaluate the surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where S is the part of the ellipsoid $x^2 + y^2 + \frac{1}{4}z^2 = 1$ for which $z \geq 0$, oriented so that its normal vector has a positive z -component.

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Problem 10 of 10 [10 points]

Which of the following statements are true (T) and which are false (F)? You do not need to give reasons. This problem will be graded by answer only. [1 point each]

- (1.) If a smooth curve C is parameterized by $\mathbf{r}(s)$ where s is arc length, then the tangent vector $\mathbf{r}'(s)$ satisfies $|\mathbf{r}'(s)| = 1$.
- (2.) If $\mathbf{r}(t)$ defines a smooth curve C in space that has constant curvature $\kappa > 0$, then C is part of a circle with radius $1/\kappa$.
- (3.) Suppose \mathbf{F} is a continuous vector field with open domain D . If

$$\int_C \mathbf{F} \cdot d\mathbf{r} = 0$$

for every piecewise smooth closed curve C in D , then \mathbf{F} is conservative.

- (4.) Suppose \mathbf{F} is a vector field with open domain D , and the components of \mathbf{F} have continuous partial derivatives. If $\nabla \times \mathbf{F} = 0$ everywhere on D , then \mathbf{F} is conservative.
- (5.) The curve defined by

$$\mathbf{r}_1(t) = \cos(t^2) \mathbf{i} + \sin(t^2) \mathbf{j} + 2t^2 \mathbf{k}, \quad -\infty < t < \infty,$$

is the same as the curve defined by

$$\mathbf{r}_2(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + 2t \mathbf{k}, \quad -\infty < t < \infty.$$

- (6.) The curve defined by

$$\mathbf{r}_1(t) = \cos(t^2) \mathbf{i} + \sin(t^2) \mathbf{j} + 2t^2 \mathbf{k}, \quad 0 \leq t \leq 1,$$

is the same as the curve defined by

$$\mathbf{r}_2(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + 2t \mathbf{k}, \quad 0 \leq t \leq 1.$$

- (7.) Suppose $\mathbf{F}(x, y, z)$ is a vector field whose components have continuous second order partial derivatives. Then $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.
- (8.) Suppose the real valued function $f(x, y, z)$ has continuous second order partial derivatives. Then $\nabla \cdot (\nabla f) = 0$.
- (9.) The region $D = \{ (x, y) \mid x^2 + y^2 > 1 \}$ is simply connected.
- (10.) The region $D = \{ (x, y) \mid y - x^2 > 0 \}$ is simply connected.