The University of British Columbia
Final Examination - December 7th, 2005

Mathematics 317
Instructor: Jim Bryan

Closed book examination — Time: 2.5 hours

Name ______________________ **Signature** ______________________

Student Number ______________

Special Instructions:
- Be sure that this examination has 12 pages. Write your name on top of each page.
- No calculators or notes are permitted.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

• Each candidate should be prepared to produce her/his library/AMS card upon request.
• No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
• Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
• CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
(a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.
(b) Speaking or communicating with other candidates.
(c) Purposely exposing written papers to the view of other candidates.

1		10
2		20
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		100

Problem 1. (10 points.) Let $\mathbf{r}(t)$ be a vector valued function. Let $\mathbf{r}'$, $\mathbf{r}''$, and $\mathbf{r}'''$ denote $\frac{d\mathbf{r}}{dt}$, $\frac{d^2\mathbf{r}}{dt^2}$, and $\frac{d^3\mathbf{r}}{dt^3}$ respectively. Express

$$\frac{d}{dt}\left[(\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}''\right]$$

in terms of $\mathbf{r}$, $\mathbf{r}'$, $\mathbf{r}''$, and $\mathbf{r}'''$. Select the correct answer.

1. $(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r}'''$
2. $(\mathbf{r}' \times \mathbf{r}'') \cdot \mathbf{r} + (\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}'''$
3. $(\mathbf{r} \times \mathbf{r}') \cdot \mathbf{r}'''$
4. $\mathbf{0}$
5. None of the above.

Problem 2. (**2 points each.**) Say whether the following statements are true (**T**) or false (**F**). You may assume that all functions and vector fields are defined everywhere and have derivatives of all orders everywhere. You do not need to give reasons; this problem will be graded by answer only.

1. The divergence of $\nabla \times \mathbf{F}$ is zero, for every $\mathbf{F}$.
2. In a simply connected region, $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on the endpoints of C.
3. If $\nabla f = \mathbf{0}$, then f is a constant function.
4. If $\nabla \times \mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is a constant vector field.
5. If $\operatorname{div} \mathbf{F} = 0$, then $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$ for every closed surface S.
6. If $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed curve C, then $\nabla \times \mathbf{F} = \mathbf{0}$.
7. If $\mathbf{r}(t)$ is a path in three space with constant speed $|\mathbf{v}(t)|$, then the acceleration is perpendicular to the tangent vector, i.e. $\mathbf{a} \cdot \mathbf{T} = 0$.
8. If $\mathbf{r}(t)$ is a path in three space with constant curvature κ, then $\mathbf{r}(t)$ parameterizes part of a circle of radius $1/\kappa$.
9. Let $\mathbf{F}$ be a vector field and suppose that S_1 and S_2 are oriented surfaces with the same boundary curve C, and C is given the direction that is compatible with the orientations of S_1 and S_2. Then $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S_1} = \iint_{S_2} \mathbf{F} \cdot d\mathbf{S_2}$.
10. Let $A(t)$ be the area swept out by the trajectory of a planet from time t_0 to time t. Then $\frac{dA}{dt}$ is constant.

Problem 3. (**10 points.**)
Find the speed of a particle with the given position function

$$\mathbf{r}(t) = 5\sqrt{2}t\mathbf{i} + e^{5t}\mathbf{j} - e^{-5t}\mathbf{k}$$

Select the correct answer:

1. $|\mathbf{v}(t)| = (e^{5t} + e^{-5t})$
2. $|\mathbf{v}(t)| = \sqrt{10 + 5e^{t} + 5e^{-t}}$
3. $|\mathbf{v}(t)| = \sqrt{10 + e^{10t} + e^{-10t}}$
4. $|\mathbf{v}(t)| = 5(e^{5t} + e^{-5t})$
5. $|\mathbf{v}(t)| = 5(e^{t} + e^{-t})$

Problem 4. (**10 points.**) Find the correct identity, if f is a function and $\mathbf{G}$ and $\mathbf{F}$ are vector fields. Select the true statement.

1. $\operatorname{div}(f\mathbf{F}) = f\,\operatorname{curl}(\mathbf{F}) + (\nabla f) \times \mathbf{F}$
2. $\operatorname{div}(f\mathbf{F}) = f\,\operatorname{div}(\mathbf{F}) + \mathbf{F} \cdot \nabla f$
3. $\operatorname{curl}(f\mathbf{F}) = f\,\operatorname{div}(\mathbf{F}) + \mathbf{F} \cdot \nabla f$
4. None of the above are true.

Problem 5. (**10 points.**) Let S be the part of the paraboloid $z+x^2+y^2=4$ lying between the planes $z=0$ and $z=1$. For each of the following, indicate with a **yes** or a **no** whether it correctly parameterizes the surface S. You **do not** need to give reasons; only the **yes/no** answer will be graded.

$$\mathbf{r}(u,v) = u\mathbf{i} + v\mathbf{j} + (4-u^2-v^2)\mathbf{k}, \quad (u,v) \in \{0 \le u^2+v^2 \le 1\}$$

$$\mathbf{r}(u,v) = (\sqrt{4-u}\cos v)\mathbf{i} + (\sqrt{4-u}\sin v)\mathbf{j} + u\mathbf{k}, \quad (u,v) \in \{0 \le u \le 1, 0 \le v \le 2\pi\}$$

$$\mathbf{r}(u,v) = (u\cos v)\mathbf{i} + (u\sin v)\mathbf{j} + (4-u^2)\mathbf{k}, \quad (u,v) \in \{\sqrt{3} \le u \le 2, 0 \le v \le 2\pi\}$$

Problem 6. (**10 points.**) Let S be the part of the plane

$$x + y + z = 2$$

that lies in the first octant oriented so that $\mathbf{N}$ has a positive $\mathbf{k}$ component. Let

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

Evaluate the flux integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$

Problem 7. (**10 points.**) Consider the vector field $\mathbf{F}(x, y, z) = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$.

1. Compute $\operatorname{curl} \mathbf{F}$.
2. If C is any path from $(0, 0, 0)$ to (a_1, a_2, a_3) and $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, show that $\int_C \mathbf{F} \cdot d\mathbf{r} = \mathbf{a} \cdot \mathbf{a}$.

Problem 8. (10 points.)
Let

$$\mathbf{F} = x \sin y \mathbf{i} - y \sin x \mathbf{j} + (x - y) z^2 \mathbf{k}.$$

Use Stoke's theorem to evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

along the path consisting of the straight line segments successively joining the points $P_0 = (0,0,0)$ to $P_1 = (\pi/2, 0, 0)$ to $P_2 = (\pi/2, 0, 1)$ to $P_3 = (0,0,1)$ to $P_4 = (0, \pi/2, 1)$ to $P_5 = (0, \pi/2, 0)$, and back to $(0,0,0)$.

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Problem 9. (**10 points.**) Let S be the hemisphere $\{x^2+y^2+z^2=1, z \geq 0\}$ oriented with $\mathbf{N}$ pointing away from the origin. Evaluate the flux integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

where

$$\mathbf{F} = (x+\cos(z^2))\mathbf{i} + (y+\ln(x^2+z^5))\mathbf{j} + \sqrt{x^2+y^2}\mathbf{k}.$$

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