Marks

[12] **1.** Let C be the osculating circle to the helix $\mathbf{r}(t) = (\cos t, \sin t, t)$ at the point where $t = \pi/6$. Find:

(a) the radius of curvature of C;

(b) the center of C;

(c) the unit normal vector to the plane of C.

Continued on page 3

[13] **2.** Let $\mathbf{F} = (yz\cos x, z\sin x + 2yz, y\sin x + y^2 - \sin z)$ and let C be the line segment $\mathbf{r}(t) = (t, t, t)$, for $0 \le t \le \pi/2$. Evaluate $\displaystyle\int_C \mathbf{F} \bullet d\mathbf{r}$.

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[13] **3.** Let S be the ellipsoid $x^2 + 2y^2 + 3z^2 = 16$, and $\mathbf{n}$ its outer unit normal.

(a) Find $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$ if $\mathbf{F}(x,y,z) = \dfrac{(x,y,z) - (2,1,1)}{[(x-2)^2 + (y-1)^2 + (z-1)^2]^{3/2}}$.

(b) Find $\iint_S \mathbf{G} \bullet \mathbf{n}\, dS$ if $\mathbf{G}(x,y,z) = \dfrac{(x,y,z) - (3,2,2)}{[(x-3)^2 + (y-2)^2 + (z-2)^2]^{3/2}}$.

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[13] **4.** Find the net flux $\iint_S \mathbf{F} \bullet \mathbf{n}\, dS$ of the vector field $\mathbf{F}(x, y, z) = (x, y, z)$ upwards (with respect to the z-axis) through the surface S parameterized by $\mathbf{r} = (uv^2, u^2v, uv)$, for $0 \leq u \leq 1$, $0 \leq v \leq 3$.

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[13] **5.** Let $\mathbf{F} = (\sin x^2, xz, z^2)$. Evaluate $\oint_C \mathbf{F} \bullet d\mathbf{r}$ around the curve C of intersection of the cylinder $x^2 + y^2 = 4$ with the surface $z = x^2$, traversed counter clockwise as viewed from high on the z-axis.

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[13] **6.** Explain how one deduces the differential form

$$\nabla \times \mathbf{E} = -\frac{1}{c}\,\frac{\partial \mathbf{H}}{\partial t}$$

of Faraday's law from its integral form

$$\oint_C \mathbf{E} \bullet d\mathbf{r} = -\frac{1}{c}\,\frac{d}{dt}\iint_S \mathbf{H} \bullet \mathbf{n}\, dS.$$

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[13] **7.** Let $\Omega \subset R^3$ be a smoothly bounded domain, with boundary $\partial\Omega$ and outer unit normal $\mathbf{n}$. Prove that for any vector field $\mathbf{F}$ which is continuously differentiable in $\Omega \cup \partial\Omega$,

$$\iiint_{\Omega} \nabla \times \mathbf{F} \, dV = -\iint_{\partial\Omega} \mathbf{F} \times \mathbf{n} \, dS.$$

Hint: Recall the fundamental lemma used in proving the divergence theorem.

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[10] **8.** True or False. Explanations are not required. Consider vector fields $\mathbf{F}$ and scalar functions f and g which are defined and smooth in all of three-dimensional space. Let $\mathbf{r} = (x, y, z)$ represent a variable point in space, and let $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$ be a constant vector. Let Ω be a smoothly bounded domain with outer unit normal $\mathbf{n}$. Which of the following are identities, always valid under these assumptions?

(a) $\nabla \bullet \nabla f = 0$

(b) $\mathbf{F} \times \nabla f = f \nabla \bullet \mathbf{F}$

(c) $\nabla^2 f = \nabla(\nabla \bullet f)$

(d) $\nabla \times \nabla f = \mathbf{0}$

(e) $(\nabla \times f) + (\nabla \times g) = \nabla f \times \nabla g$

(f) $\nabla \bullet \nabla \times \mathbf{F} = \mathbf{0}$

(g) $\nabla \bullet \dfrac{\mathbf{r}}{|\mathbf{r}|^2} = 0$, for $\mathbf{r} \neq \mathbf{0}$

(h) $\nabla \times (\boldsymbol{\omega} \times \mathbf{r}) = \mathbf{0}$

(i) $\displaystyle\iiint_\Omega f \nabla \bullet \mathbf{F}\, dV = -\iiint_\Omega \nabla f \bullet \mathbf{F}\, dV + \iint_{\partial\Omega} f\mathbf{F} \bullet \mathbf{n}\, dS$

(j) $\displaystyle\iint_{\partial\Omega} f\mathbf{n}\, dS = -\iiint_\Omega \nabla f\, dV$

The End

Be sure that this examination has 9 pages including this cover

The University of British Columbia

Final Examinations - April, 2005

Mathematics 317

Section 201

J. Heywood

Closed book examination | Time: 2.5 hours

Name ______________________ **Signature** ______________________

Student Number ______________________ **Instructor's Name** ______________________

Section Number ______________________

Special Instructions:

Calculators and books are not permitted.

One $8\frac{1}{2}$" $\times$ 11" two-sided page of notes is permitted.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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