The University of British Columbia

Final Exam
Math 316, Sect 201, April 12, 2006

Name (print): ____________________
Student No.: ____________________

Closed book examination. Time 2.5 hours.
There are five questions worth a total of 100 marks.
No notes allowed.
Non-programming calculator is not needed (but is allowed).
One self-made formula sheet can be used.
In all questions, you must show work — i.e. display intermediate results — to get full credit.
You may use one letter sized formula sheet and a calculator.
Be neat! I will not attempt to decipher messy calculations.

Rules governing examination

- Each candidate should be prepared to produce his/her own library/AMS card upon request.
- No candidates shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
- Candidates are not permitted to ask questions to invigilators, except in cases of supposed errors or ambiguities in examination questions.
- CAUTION — Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action: (a) Making use of any books, papers or memoranda, other then those authorized by the examiner; (b) speaking or communicating with other candidates; (c) purposely exposing written papers to the view of other candidates.
- Smoking is not permitted during examination.

Question	1	2	3	4	5	6	
Mark							
Total	8	14	16	16	22	24	100

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Problem 1. Find all singular points of the given equation and determine whether each one is regular or irregular.

$$(x^2 + x - 2)y'' + (x + 1)y' + 2y = 0.$$

Problem 2. Consider the ODE

$$4xy'' + 2y' + y = 0.$$

1. Verify that $x = 0$ is a regular singular point.
2. Find the indicial equation.
3. Find the recurrence relation.
4. Find the first three non-zero terms of the both linearly independent series solutions.
5. Find the radii of convergence of the series solutions.

Problem 3. Consider a bar of length 1 whose left end is kept at zero degrees and whose right end is insulated, with some initial heat distribution $f(x)$. State the initial-boundary value problem appropriate for this situation. Find the temperature distribution in the bar at any time t. (Take $c^2 = 1$.)

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Problem 4. Solve the initial boundary value for the wave equation $u_{tt} = c^2 u_{xx}$, $0 < x < 1$, $t > 0$ with boundary conditions $u(0, t) = u(1, t) = 0$ and initial conditions $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$, where

$$f(x) = 2 \sin 2\pi x + 3 \sin 3\pi x, \qquad g(x) = \sin \pi x.$$

Problem 5. Solve the 2-dimensional heat problem with u vanishing on the boundary of rectangle $0 \le x \le a$, $0 \le y \le b$ and given initial heat distribution

$$f(x,y) = 8 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} - \sin \frac{2\pi x}{a} \sin \frac{3\pi y}{b}.$$

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Problem 6. Solve the vibrating problem for circular membrane of radius 1 with $c = 1$, clamped along its circumference for the given initial data

$$u(r, \theta, 0) = J_3(\alpha_{32r}) \sin 3\theta, \qquad u_t(r, \theta, 0) = 0,$$

where α_{32} is the second positive zero of function J_3.

The end