100pts; 2.5 hrs

I Short answer questions: Each question carries 6 marks, your answers should quote the results being used and show your work.

1) Find two integers congruent to 3 mod 5 and 4 mod 7.

2) For which positive integers m will we have $1000 \equiv 1 \text{mod } m$?

3) Find the least positive residue of $1! + 2! + \cdots + 100!$.

4) Suppose that $n = 81294358X$. Write down a digit in the slot marked X so that n is divisible by a) 11 b) 9 c) 4.

5) Find all solutions of $7x \equiv 4 \text{ mod } 13$.

6) Define a Carmichael number. Use the necessary and sufficient condition for a number to be a Carmichael number to show that 561 is a Carmichael number.

7) What is the remainder when 5^{16} is divided by 23?

II State whether the following are true or false with full justification. Each question carries 4 marks.

1) If a positive integer has exactly 3 positive divisors, then it is necessarily of the form p^2 where p is a prime.

2) The order $ord_1 9(5)$ is 7.

3) The number 25 passes Miller's test for the base 7.

4) The number $2^{39} - 1$ is divisible by 7.

III Find all positive integers n such that $n!$ ends with exactly 74 zeros in decimal notation. 14marks

IV Define the sum of divisors function $\sigma(n)$ and number of divisors function $\tau(n)$. Show that there is no positive integer n with $\phi(n) = 14$.

V Show that if a and b are relatively prime integers, then $a^{\phi(b)} + b^{\phi(a)} \equiv 1$ mod ab. Find the inverse of What is the multiplicative inverse of 5^8 modulo 16?