

1. TRUE OR FALSE

1. (7 points) Suppose n is an integer with exactly 3 positive divisors. Then $n = p^2$ for some prime p .

A. True, B. False.

2. (7 points) Suppose that $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ with $c|a$ and $d|b$. Then

$$\frac{a}{c} \equiv \frac{b}{d} \pmod{m}.$$

A. True, B. False.

2. MULTIPLE CHOICE

3. (7 points) The number of primitive roots of 98 is

- A. 0, B. 12, C. 34, D. 42, E. None of the above.

4. (7 points) The number of primitive roots of 99 is

- A. 0, B. 8, C. 34, D. 66, E. None of the above.

5. (7 points) The number of zeros at the end of the decimal representation of $153!$ is

A. 28, B. 33, C. 37, D. 62, E. None of the above.

6. (7 points) $10^{200,000,000,000}$ days from today it will be

A. Sunday, B. Monday, C. Tuesday, D. Wednesday, E. None of the above.

7. (10 points) Let n be the solution to the following ancient Indian problem (taken from Rosen):

If eggs are removed from a basket 2, 3, 4, 5 and 6 at a time, there remain respectively, 1, 2, 3, 4 and 5 eggs. But if the eggs are removed 7 at a time, no eggs remain. What is the least number of eggs that could have been in the basket?

The number n is congruent to which of the following modulo 13?

- A. 1, B. 2, C. 3, D. 4, E. None of the above.

3. PROVE OR DISPROVE

For this problem, *clearly indicate* whether the statement is *true or false* then prove or disprove it.

8. (12 points) If $\phi(n)|n - 1$ then n is squarefree. (Here ϕ is the Euler ϕ -function.)

4. PROVE

In this section, prove the statement given to you.

9. (12 points) Suppose p is a prime with $p \equiv 1 \pmod{4}$, and r is a primitive root modulo p . Show that $-r$ is also a primitive root modulo p .

10. (12 points) Suppose a and b are positive integers. Show that

$$(a, b)[a, b] = ab.$$

11. (12 points) Suppose a and N are integers with $N \geq 0$. Show that

$$(1 + a)^N \equiv 1 + Na \pmod{a^2}.$$

Math 312

Final

Fall 2005

Patrick Brosnan, Instructor

Name: _____

Student ID Number: _____

Section: _____

Signature:

By signing here, you confirm you are the person identified above and that all the work herein is solely your own.

Instructions:

- You are allowed to use pencil, pen and eraser only. **No notes, index cards or calculators**
- You may use the back of a sheet for calculations.
- Put your name on **all** sheets in the allotted space.
- **Box any final answers.**

Problem	Points	Score
1	7	
2	7	
3	7	
4	7	
5	7	
6	7	
7	10	
8	12	
9	12	
10	12	
11	12	
Total	100	