# Final Exam

- This exam consists of **8 questions** on **12 pages** worth a total of 100 points.
- Make sure this exam has **12 pages**.
- Duration: 2.5 hours.
- Enter your final answer in the box when provided.
- You may use a scientific calculator. Notes or other electronic devices are not allowed.
- The last page is empty for additional space. If a question is solved there, mark it clearly or it may not be graded.

First Name: ______________________ Last Name: ______________________

Student No.: ______________________ Signature: ______________________

## Student conduct during examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   (i) speaking or communicating with other examination candidates, unless otherwise authorized;
   (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
   (iii) purposely viewing the written papers of other examination candidates;
   (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

| Question | Points | Score |
|---|---|---|
| 1 | 15 | |
| 2 | 8 | |
| 3 | 14 | |
| 4 | 11 | |
| 5 | 14 | |
| 6 | 16 | |
| 7 | 11 | |
| 8 | 11 | |
| Total: | 100 | |

Common Distributions

| Distribution | p.m.f. / p.d.f. | Mean | Variance |
|---|---|---|---|
| $\mathrm{Bin}(n,p)$ | $\binom{n}{k}p^k(1-p)^{n-k}$ | $np$ | $np(1-p)$ |
| $\mathrm{Geom}(p)$ | $p(1-p)^{k-1}$ | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ |
| NegBin(n,p) | $\binom{n+k-1}{n-1}p^n(1-p)^k$ | $\frac{n}{p}$ | $n\frac{1-p}{p^2}$ |
| $\mathrm{Poisson}(\lambda)$ | $e^{-\lambda}\frac{\lambda^k}{k!}$ | $\lambda$ | $\lambda$ |
| $\mathrm{Uniform}(a,b)$ | $\frac{1}{b-a}$ | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ |
| $\mathrm{Exp}(\lambda)$ | $\lambda e^{-\lambda x}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ |
| $N(\mu,\sigma^2)$ | $\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}$ | $\mu$ | $\sigma^2$ |

The probability that $\mathbb{P}(0 < Z < x)$, where $Z \sim N(0,1)$:

| $x$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |

15 marks 1. (a) Define the moment generating function of a random variable $X$.

(b) Define the covariance of two random variables $X$, $Y$.

(c) Define that the events $A$ and $B$ are independent.

(d) State the Chebyshev Inequality.

(e) State the Central Limit Theorem.

8 marks

2. Assume that $X, Y$ are independent random variables such that $\mathbb{E}(X) = 1$, $\operatorname{Var}(X) = 2$, $\mathbb{E}(Y) = 3$, and $\operatorname{Var}(Y) = 4$.

   (a) Calculate $\mathbb{E}(2X + 4XY - Y)$.

   Answer:

   (b) Find $\operatorname{Var}(5X + 3Y)$.

   Answer:

14 marks

3. A die is tossed seven times.

   (a) What is the probability that exactly two outcomes are 1?

   Answer:

   (b) What is the probability that all outcomes are odd, given that the first outcome was greater than 3?

   Answer:

11 marks 4. Let $X$, $Y$ be independent exponential random variables with the same parameter $\lambda$. Find the probability density function of $X - Y$.

Answer:

14 marks

5. A random variable $X$ has moment generating function $g_X(t) = (1-3t)^{-2}$ for $t < 1/3$.

   (a) Find $\mathbb{E}(X)$.

   Answer:

   (b) Find $\mathrm{Var}(X)$.

   Answer:

16 marks

6. Assume that the random variables $X, Y$ have joint density function

$$f(x,y) = \begin{cases} c(3x^2 + 2y) & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the constant $c$.

Answer:

(b) Find the marginal probability density function of $Y$.

Answer:

(c) Find the conditional expectation $\mathbb{E}(X \mid Y = 1)$.

Answer:

11 marks 7. Let $X$ and $Y$ be independent geometric random variables with the same parameter $p$. What is the probability that $X + Y$ is odd?

Answer:

11 marks

8. Let $S_{200}$ be the number of heads that turn up in 200 tosses of a fair coin. Use the Central Limit Theorem to estimate the probability $\mathbb{P}(90 \leq S_{200} \leq 110)$.

Answer:

*This page left blank for your workings and solutions. Problems solved:* ______