

Be sure this exam has 10 pages including the cover

The University of British Columbia

Sessional Exams – 2009 Winter Term 1
Mathematics 302 Introduction to Probability

Name: _____

Student Number: _____ Section: _____

This exam consists of 8 questions worth 10 marks each. No aids other than calculators are permitted.

Problem	total possible	score
1.	10	
2.	10	
3.	10	
4.	10	
5.	10	
6.	10	
7.	10	
8.	10	
total	80	

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- (a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
- (b) Speaking or communicating with other candidates.
- (c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

Tables on last page.

1. A US scientist has 12 ties on a tie-rack. Four are red, four are white and four are blue. Each day he wears one of the twelve selected at random and at the end of the day returns it to the tie-rack.

(4 points)

- (a) What is the probability that none of the ties he wears next week are red?

(3 points)

- (b) What is the probability that not all the colours red white and blue are used next week?

(3 points)

- (c) If, instead of returning each tie to the tie-rack at the end of the day he gives it away to his lab assistant, what is the probability that none of the ties he wears next week are red?

- (10 points) 2. Marie is getting married on Dec 5. In recent years, the probability of rain on Dec 5 has been $\frac{1}{73}$. Unfortunately, the forecast predicts it will rain for Dec 5. When it actually rains, the forecast correctly predicts rain 90% of the time. When it doesn't rain, rain is still forecast 10% of the time. What is the probability that it will rain on Dec 5?

3. The organisers of a banquet at a restaurant have ordered 100 place settings, but they have invited 103 people. Past experience indicates that 3% of the people who are invited independently decide not to come.

(7 points)

- (a) What is the approximate probability that exactly 100 people come?

(3 points)

- (b) If more than 100 people come there will be a surcharge of \$100 times the number of additional people. What is the expected surcharge?

4. Each light on a Christmas tree is immediately replaced when it fails. The failures for each light constitute a Poisson process with rate $1/500$ failures per hour. There are twenty lights on the tree.

(2 points)

- (a) What is the probability that the light at the top of the tree lasts for all 12 days of Christmas?

(4 points)

- (b) What is the probability that at least one light fails on Dec 25?

(2 points)

- (c) If at least two lights failed in the last 24 hours what is the probability that exactly two will fail in the next 24 hours?

(2 points)

- (d) If at least two lights failed in the last 24 hours what is the probability that exactly three failed in the last 24 hours?

5. A continuous random variable X has probability density function

$$f(x) = \begin{cases} cx & \text{if } 0 \leq x \leq 1 \\ x^2/7 & \text{if } 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

(5 points) (a) Find the value of c .

(5 points) (b) Find x such that $P(X \geq x) = 1/21$.

6. The mass of a tribble has a normal distribution with mean 400 grams, and 80.3% of all tribbles have a mass between 390 and 410 grams.

(5 points)

- (a) Find the standard deviation of the mass of a tribble.

(5 points)

- (b) How many tribbles must be taken in a sample so that with probability 0.803 the average mass of the tribbles in the sample is between 399 and 401 grams?

7. Two contractors, A and B, bid independently on a job. The contract will go to the lowest bidder. A's bid is a random number X selected (with uniform distribution) from the interval $[1, 2]$, while B's bid Y has probability density function

$$f_Y(y) = \begin{cases} \frac{2}{3}y & \text{for } 1 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (2 points) (a) What is the joint probability density function of X and Y ?

- (4 points) (b) What is the probability that A wins the contract, i.e. that $X < Y$?

- (4 points) (c) What is the expected value of the winning bid (i.e. $E[\min(X, Y)]$)?
Hint: $\min(X, Y) = X$ if $X < Y$ and Y if $Y < X$.

8. Let X_1, X_2, \dots, X_{50} be independent random variables, each with the same probability density function

$$f(x) = \begin{cases} x/2 & \text{for } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

and $\bar{X} = (X_1 + \dots + X_{50})/50$ their average.

(3 points)

- (a) Find the variance of \bar{X} .

(3 points)

- (b) Find an upper bound for the probability that \bar{X} deviates from its expected value by more than 0.1, i.e. $|\bar{X} - E(\bar{X})| > 0.1$.

(4 points)

- (c) Find a good approximation for the probability that \bar{X} deviates from its expected value by more than 0.1.

Table 1: Mean and Variances

Distribution	Mean	Variance
Bin (n, p)	np	$np(1 - p)$
Geometric (p)	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson (λ)	λ	λ
Uniform (a, b)	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp (λ)	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Table 2: cdf of normal distribution

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990