Be sure this exam has 8 pages including the cover

The University of British Columbia

MATH 301, Sections 201

Final Exam – April 15, 2016, 07:00PM-09:30PM, BUCH B213

Name ______________________ **Signature** ______________________

Student Number ______________

This exam consists of **7** questions. No notes. Write your answer in the blank page provided.

Problem	max score	score
1.	14	
2.	10	
3.	20	
4.	6	
5.	20	
6.	20	
7.	10	
total	100	

(14 points) 1. Use complex contour to compute the following real integral

$$\int_0^{+\infty} \frac{\sqrt{x}\ \log(x)}{x^2+1} dx$$

(10 points) 2. Find the image of $\mathbf{C}\backslash\{-1 \leq x \leq 1,\ y = 0\}$ under the mapping $w = (\frac{z-1}{z+1})^{\frac{1}{3}}$, where $\mathbf{C}$ is the complex plane. Here we take the principal branch of the cubic root.

Hint: do it in steps.

(20 points) 3. Using conformal mapping to find the solution $\phi(x, y)$ to Laplace's equation outside the two circles $C_1 : |z - 2i| = 1$ and $C_2 : |z + 2i| = 1$. You are given that $\phi = 0$ on C_1 and $\phi = 1$ on C_2.

Hint: You may assume that the symmetric points are on the imaginary axis.

(6 points) 4. Let $\Omega(z) = i(z^2 - 1)^{\frac{1}{2}} = \Phi(z) + i\Psi(z)$ be a complex velocity potential, where the principal branch of square root is chosen. Find the streamline $\Psi(z) = 0$.

(20 points) 5. Using Fourier Transform to find an integral representation for the solution to

$$u_{tt} = -u_{xxxx}, -\infty < x < \infty, t > 0$$

$$u(x,0) = f(x),\ u_t(x,0) = 0$$

We may assume that $u, u_x, u_{xx}, u_{xxx} \to 0$ as $|x| \to +\infty$. Write your solution in terms of a convolution with $f(x)$.

Hint: You may use the following fact

$$\int_0^{+\infty} e^{-ix^2}\, dx = \frac{\sqrt{\pi}}{2\sqrt{2}}(1-i)$$

(20 points) 6. Using the Laplace Transform to solve the following first order differential equation

$$y^{''} + 5y^{'} + 6y = f(t), t > 0, \ y(0) = 1, y^{'}(0) = 0$$

where $f(t) = e^{-t}$ for $0 \leq t \leq 1$ and $f(t+1) = f(t)$ for $t > 1$. Find the periodic part and non-periodic part of the solution respectively.

Hint: You may use the following fact

$$\mathcal{L}[u_c(t)f(t-c)](s) = e^{-cs}\mathcal{L}[f](s)$$

where $\mathcal{L}$ denotes the Laplace transform and $u_c(t) = 0$ for $t < c$ and $u_c(t) = 1$ for $t > c$.

(10 points) 7. Using the Nyquist Theorem to show that the solution to

$$y^{'''} + y^{''} + 4y^{'} + y = e^{-t}, \quad y(0) = 1, y^{'}(0) = 0, y^{''}(0) = -1$$

approaches to zero as $t \to +\infty$.