

Be sure this exam has 8 pages including the cover

The University of British Columbia

MATH 301, Sections 201

Final Exam – April 15, 2016, 07:00PM-09:30PM, BUCH B213

Name _____ Signature _____

Student Number _____

This exam consists of 7 questions. No notes. Write your answer in the blank page provided.

Problem	max score	score
1.	14	
2.	10	
3.	20	
4.	6	
5.	20	
6.	20	
7.	10	
total	100	

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

(14 points) 1. Use complex contour to compute the following real integral

$$\int_0^{+\infty} \frac{\sqrt{x} \log(x)}{x^2 + 1} dx$$

- (10 points) 2. Find the image of $\mathbf{C} \setminus \{-1 \leq x \leq 1, y = 0\}$ under the mapping $w = \left(\frac{z-1}{z+1}\right)^{\frac{1}{3}}$, where \mathbf{C} is the complex plane. Here we take the principal branch of the cubic root.

Hint: do it in steps.

- (20 points) 3. Using conformal mapping to find the solution $\phi(x, y)$ to Laplace's equation outside the two circles $C_1 : |z - 2i| = 1$ and $C_2 : |z + 2i| = 1$. You are given that $\phi = 0$ on C_1 and $\phi = 1$ on C_2 .
Hint: You may assume that the symmetric points are on the imaginary axis.

- (6 points) 4. Let $\Omega(z) = i(z^2 - 1)^{\frac{1}{2}} = \Phi(z) + i\Psi(z)$ be a complex velocity potential, where the principal branch of square root is chosen. Find the streamline $\Psi(z) = 0$.

(20 points) 5. Using Fourier Transform to find an integral representation for the solution to

$$u_{tt} = -u_{xxxx}, \quad -\infty < x < \infty, t > 0$$

$$u(x, 0) = f(x), \quad u_t(x, 0) = 0$$

We may assume that $u, u_x, u_{xx}, u_{xxx} \rightarrow 0$ as $|x| \rightarrow +\infty$. Write your solution in terms of a convolution with $f(x)$.

Hint: You may use the following fact

$$\int_0^{+\infty} e^{-ix^2} dx = \frac{\sqrt{\pi}}{2\sqrt{2}}(1 - i)$$

(20 points) 6. Using the Laplace Transform to solve the following first order differential equation

$$y'' + 5y' + 6y = f(t), t > 0, y(0) = 1, y'(0) = 0$$

where $f(t) = e^{-t}$ for $0 \leq t \leq 1$ and $f(t+1) = f(t)$ for $t > 1$. Find the periodic part and non-periodic part of the solution respectively.

Hint: You may use the following fact

$$\mathcal{L}[u_c(t)f(t-c)](s) = e^{-cs}\mathcal{L}[f](s)$$

where \mathcal{L} denotes the Laplace transform and $u_c(t) = 0$ for $t < c$ and $u_c(t) = 1$ for $t > c$.

(10 points) 7. Using the Nyquist Theorem to show that the solution to

$$y''' + y'' + 4y' + y = e^{-t}, \quad y(0) = 1, y'(0) = 0, y''(0) = -1$$

approaches to zero as $t \rightarrow +\infty$.