The University of British Columbia

Final Examination - April, 2011

Mathematics 301

Closed book examination Time: 2.5 hours

Last Name: ________________ , **First:** ____________ **Signature** ________________

Student Number ________________

Special Instructions:

No books, notes or calculators are allowed.

Include explanations and simplify answers to obtain full credit.

Use backs of sheets if necessary.

The last page is for scrap work - tear it off and do not hand it in.
It will not be marked.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		15
2		17
3		17
4		17
5		17
6		17
Total		100

1. [15]

(a) Compute all values of $(-i)^{1+i}$

(b) Find all solutions, z, of $\cos(z) = ki$ where k is a positive real number.

2. [17]
Evaluate the integral, I, explaining clearly the choice of the contour and all details of the calculation

$$I = \int_0^\infty \frac{x^{\frac{1}{2}}\,dx}{x^2 - x + 1}$$

3. [17]

(a) $g(z)$ is defined as $g(z) = (1 - z^2)^{\frac{1}{2}}$ with a finite branch cut for $y = 0$, $-1 < x < 1$, and $g(i) > 0$. Find $g(1 - 2i)$.

(b) Evaluate I, carefully explaining all steps:

$$I = \int_{-1}^{1} \frac{(1-x^2)^{\frac{1}{2}}\,dx}{x^2+1}.$$

4. [17]
(a) Sketch the region $D : \{\mathrm{Re}\, z > 0\} \cap \{|z-1| > 1\}$
(b) Solve: $\nabla^2\phi = 0$ in D with

$$\begin{aligned} \phi &= 1 \text{ on } \mathrm{Re}\, z = 0, \\ \phi &= 5 \text{ on } |z-1| = 1. \end{aligned}$$

(c) Hence calculate $\phi(3, 4)$.

5. [17]

The complex potential $w(z)$ for a source of strength 2π located at $z = a$ in a steady inviscid flow is $w(z) = \log(z - a)$

Consider a source of strength 2π at $z = -1$ and a sink of strength -2π at $z = 1$.

(a) Find the complex potential for the flow.

(b) Find an expression for the streamlines in the form $G(x, y) = 0$, simplified as much as possible; sketch several.

(c) Now add a uniform flow with speed V parallel to the $x-$ axis.

Find the complex potential of the new flow and calculate the location of any stagnation points.

6. [17]

(a) Find the Fourier transform of

$$f(x) = \frac{1}{(4 + x^2)}, \quad -\infty < x < \infty,$$

carefully explaining all steps. The Fourier transform and its inverse are defined as:

$$\begin{aligned} \mathcal{F}(f(x)) &= \widehat{F}(k) = \int_{-\infty}^{\infty} f(x) e^{-ixk} dx, \\ \mathcal{F}^{-1}(\widehat{F}(k)) &= f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{F}(k) e^{ixk} dk. \end{aligned}$$

(b) Solve the boundary-value problem:

$$u_t - u_{xx} + u = 0, \quad -\infty < x < \infty, \ 0 < t;$$
$$u(x,0) = g(x), \quad -\infty < x < \infty.$$

Here $|g(x)| \to 0$ as $|x| \to \infty$. You may need the result:

$$\mathcal{F}^{-1}(e^{-\alpha k^2}) = \frac{1}{2\sqrt{\alpha}} e^{-x^2/4\alpha}.$$

The final solution can be left as a real integral.

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