Be sure this exam has 9 pages including the cover

The University of British Columbia

MATH 300, Section 201

Final Exam – Apr 14, 2016

Family Name _____ **Given Name** _____

Student Number _____ **Signature** _____

This exam consists of **6** parts. No notes nor calculators. Note the number of marks for each question. Use your time wisely. **Time**: $2\frac{1}{2}$ hours

Problem	max score	score
1.	18	
2.	8	
3.	6	
4.	6	
5.	6	
6.	6	
total	50	

(18 points) 1. Do the following short answer questions. Check your answer very carefully. (Work will be considered for this problem).

(3 points) (a) Find all roots to the equation $(z+1)^{10} = z^{10}$.

(3 points) (b) Let $f(z) = \frac{1}{z^3(z-i)^2}$. Then the residue of f at $z = i$, denoted as $\text{Res}(f; i)$ is

(b) _____

(3 points) (c) Find a branch of $\log(z^2 + iz - 3)$ such that it is analytic at $z = i$, and find its derivative at $z = i$.

(3 points) (d) Compute $\int_\Gamma \frac{z-i}{z^3+4z^2}dz$ where Γ is the circle $|z| = 10$ traversed once counterclockwise.

(3 points) (e) Compute $\int_\Gamma z^{\frac{1}{2}}dz$ for the principal branch of $z^{\frac{1}{2}}$ along the line segment going from π to i.

(3 points) (f) Compute $\int_\Gamma \frac{z^7}{(2\cos z - 2 + z^2)^2} dz$ where Γ is the circle $|z| = \frac{1}{100}$ traversed once counterclockwise.

(8 points) 2. Do the following questions. You must write clearly your arguments and justify.

(4 points) (a) If a complex-valued function f is analytic in $1 \leq |z| \leq 2$, $|f(z)| \leq 3$ on $|z| = 1$, $|f(z)| \leq 12$ on $|z| = 2$, prove that $|f(z)| \leq 3|z|^2$ for all $1 \leq |z| \leq 2$.

(4 points) (b) Does there exist a function $F(z)$ analytic in the annulus $D : 1 < |z| < 2$ such that $F'(z) = 1/z$ for all $z \in D$? If yes construct such a function and justify. If no give a proof.

(6 points) 3. Find the Laurent series for the following functions in the specified domains. You must write out explicitly the first four terms.

a) $\frac{z}{z^2-z-2}$, for $1 < |z| < 2$;

b) $\frac{1}{e^z-1}$, for $0 < |z| < 2\pi$.

(6 points) 4. By using the method of contour integrals, compute $\int_0^\infty \frac{x^6}{(x^4+1)^2}dx$. (Solutions obtained by other methods will not receive any credit!)

(6 points) 5. By using the theory of residues, compute $\text{p. v.} \int_{-\infty}^{\infty} \frac{x \sin x}{x^2 - 2x + 10} dx$.

(6 points) 6. By using the theory of residues, compute $\displaystyle\int_0^{\pi} \frac{1}{(a+\sin^2\theta)^2} d\theta$ where $a > 0$.