Math 300 Final Exam

Dec 19, 2007

Duration: 150 minutes

Name: ______________________ Student Number: ______________________

Do not open this test until instructed to do so! This exam should have 13 pages, including this cover sheet. No textbooks, calculators, or other aids are allowed. One page of notes is allowed. Turn off any call phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam. **Circle your solutions! Reduce your answer as much as possible. Explain your work.** Relax. Use the back of the page if necessary.

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	**Out of**	**Score**
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	30	
Total	100	

Problem 1 (10 points)

Show that if u is harmonic in a domain D and v is a harmonic conjugate to u in D, then $u^2 - v^2$ and $2uv$ are harmonic in D.

Problem 2 (10 points)

Determine the order of the pole at $z_0 = 0$ of the functions

(a) $f(z) = \dfrac{\cos(z^2) - 1}{z^6}$, and

(b) $f(z) = \dfrac{1}{z \tan(z)}$.

Problem 3 (10 points)

Find the Laurent series expansion of $f(z) = \dfrac{4}{(1+z)(3-z)}$ around $z_0 = 0$ in the annulus $1 < |z| < 3$.

Problem 4 (10 points)

Compute the integral

$$I = \int_{C_1(0)} \frac{e^{\frac{1}{z}}}{z}\, dz,$$

where $C_1(0)$ is the circle of radius 1 around $z = 0$.

Problem 5 (10 points)

Show: If $f(z)$ is entire and $|f(z)| \to \infty$ as $|z| \to \infty$, then $f(z)$ must have at least one zero. Hint: Consider $g(z) = 1/f(z)$ and use Liouville's theorem.

Problem 6 (10 points)

Compute

$$\int_{C_2(0)} \frac{z+1}{z^2-4z+3}\,dz,$$

where $C_2(0)$ is the circle of radius 2 around $z=0$.

Problem 7 (10 points)

Compute the integral

$$I = \int_0^{2\pi} \frac{1}{2+\cos(x)}\, dx.$$

Problem 8 (30 points)

For each of the statements below, indicate whether they are true or false. **If true, give a proof. If false, give a counter example or explain why it cannot be true.**

(a) $1^z = 1$ for all $z \in \mathbb{C}$.

(b) $|e^{z^2}| \leq e^{|z|^2}$ for all $z \in \mathbb{C}$.

(c) $|e^{-z}| \leq 1$ if $|z| \leq 1$.

(d) $\max_{|z|\leq 1} |z(z-i)| = 2$.

(e) $f(z) = e^{\sin(z^2+1)}$ is infinitely many times differentiable.

(f) $\int_C \frac{z-1}{(z-2)(z-3)}\, dz = -2\pi i$ if C is the circle of radius 5 around $z = 5i$.

(g) The function $f(z) = \mathrm{Log}(z^5)$ is analytic in the complex plane except for the non-positive real axis.

(h) Let $u(x, y)$ be the real part of a function f that is analytic in a domain D. Then $\frac{\partial^k u}{\partial x^k}$ is harmonic for every positive integer k.

(i) $\mathrm{Log}(z^2) = 2\mathrm{Log}(z)$ for all $z \in \mathbb{C} \setminus \{0\}$.

(j) If f is analytic in a domain D and $\mathrm{Re} f(z) = \mathrm{Im} f(z)$ for all z in D, then f is constant in D.

Extra Page