The University of British Columbia

MATH 300

Final Examination - 2005 December 19

Sections 101,102
Instructors: Dr. Rolfsen, Dr. Angel.

Closed book examination Time: 2.5 hours

Name ______________________ **Signature** ______________________

Student Number ______________

Special Instructions:
- Be sure that this examination has 8 pages. Write your name on top of each page.
- No calculators or notes are permitted.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate should be prepared to produce her/his library/AMS card upon request.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates.

1		15
2		16
3		15
4		15
5		15
6		24
Total		100

[15pt] 1. Find all complex solutions to the equation $\sin z + \cos z = 1$.

[8pt] (2a) Find all values a, b, c so that $u(x, y) = ax^2 + bxy + cy^2$ is harmonic.
[8pt] (2b) For each such u, find its harmonic conjugate.

3. For each of the following functions, describe the singularity it has at $z_0 = 0$, find a Laurent series that converges at $z = 3$ and specify the domain of convergence for the series.

[5pt] (3a) $f(z) = \dfrac{1}{z(z+2)}$

[5pt] (3b) $g(z) = \dfrac{z - \sin z}{z^3}$

[5pt] (3c) $h(z) = (1+z)e^{1/z^2}$

4. Suppose f is analytic in the punctured disc $D = \{0 < |z| < 1\}$.

[5pt] (4a) If f has a removable singularity at $z = 0$, prove that f has an anti-derivative in D.

[5pt] (4b) If $\texttt{Res}(f, 0) = 0$, prove that f has an anti-derivative in D.

[5pt] (4c) If $\texttt{Res}(f, 0) \neq 0$, prove that f **does not** have an anti-derivative in D.

[15pt] 5. Calculate the real integral $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$ using complex integration. Indicate your reasoning clearly, including limiting arguments.

6. Calculate the following integrals

[6pt] (6a) $\oint_C (\bar{z})^2 \, dz$, where C is the circle $|z-1| = 1$, oriented counterclockwise.

[6pt] (6b) $\oint_{|z|=1} z \cos(z^{-1}) \, dz$, with the contour oriented counterclockwise.

[6pt] (6c) $\oint_{|z|=2} \frac{e^{2z}}{(z+1)^3} \, dz$, with the contour oriented counterclockwise.

[6pt] (6d) $\int_\Gamma z e^{z^2} \, dz$, where Γ is the curve in the complex plane given by $y = \sin x$ for $0 \leq x \leq \pi/2$.

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