Math 267 Final Exam
Section 101
December 10, 2011
Duration: 150 minutes

Name: ______________________ **Student Number:** ______________________

Do not open exam until so instructed.

- This exam has twenty-one pages, including this cover, and four blank pages at the end.
- One 8.5x11" double-sided page of notes is allowed. No textbooks, calculators, or other aids are allowed.
- Cell phones and other electronic devices must be switched off and left at the front of the room.
- You must remain in the exam room until the exam is over.
- Write answers in the indicated 'boxes'. Explain your work, using the back of the previous page if necessary.
- Problems are *not* organized by difficulty.
- Relax.

Pages	**Marks**	**Score**
2-3	14	
4-6	16	
7-9	16	
10-12	19	
13-15	19	
16-17	16	
Total	100	

Rules governing all UBC examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

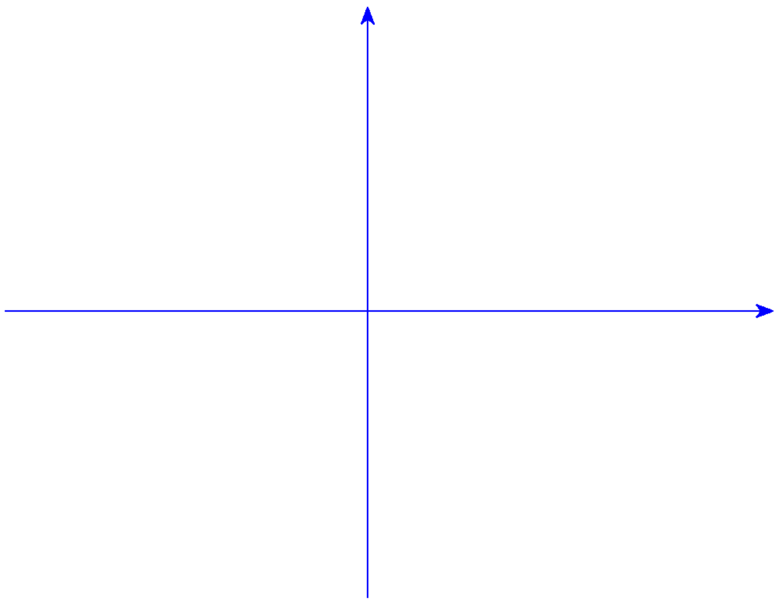
(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem 1 (4 Marks)

Plot complex numbers $\alpha = 2e^{-i\frac{\pi}{6}}$ and $\beta = e^{+i\frac{3\pi}{4}}$, in the space provided. Label axes appropriately.



Problem 2 (3 Marks)

Rewrite $\cos(x)\cos(3x)$ as a simple sum of cosines.

$\cos(x)\cos(3x) =$

Problem 3

Compute the sum. Write your answer in the space provided.

Part A (3 Marks)

$$\sum_{j=-5}^{+5} (-3)^j =$$

Part B (4 Marks)

$$\sum_{k=-\infty}^{+\infty} \left(\frac{1}{2}\right)^{|k|} =$$

Problem 4

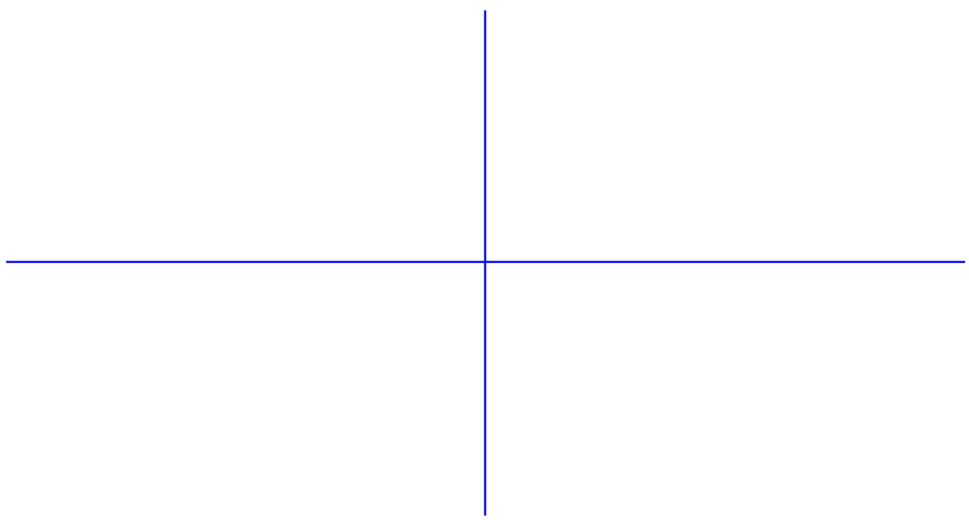
For both parts of this question, consider $g(x)$, defined only for $x \in [0, 2\pi]$,

$$g(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq \dfrac{\pi}{2} \\ 1 & \text{for } \dfrac{\pi}{2} < x < \pi \\ 0 & \text{for } \pi \leq x \leq 2\pi \end{cases}$$

Part A

(2 Marks)

For $x \in [-2\pi, 2\pi]$, sketch the odd extension $g_{\text{odd}}(x)$ in the space provided,



Part B

(6 Marks)

Compute the coefficients $c_{k,\text{odd}}$ for the Fourier series of $g_{\text{odd}}(x)$.

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(Problem 4, Part B Continued)

$$c_{k,\text{odd}} = \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

Problem 5 (8 Marks)

Solve. You may use any formula, notation or method discussed in lecture.

$$\begin{cases} \partial_t^2 u(t,x) = \partial_x^2 u(t,x) \\ u(t,0) = 0 \\ u(t,1) = 0 \\ u(0,x) = \sin(2\pi x) \\ \partial_t u(0,x) = \sin(\pi x) \end{cases}$$

$u(t,x) =$

Problem 6

Show your work. You may use any formula or shortcut discussed in lecture.

Part A (5 Marks)

Compute the Fourier transform of,

$$f(\omega) = 3\ \mathrm{sinc}(\omega + 1)$$

$\mathcal{F}\left[f(\omega)\right](t) =$

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Part B (6 Marks)

Compute the Fourier transform of,

$$g(t) = e^{it}\, e^{-t} u(t)$$

$\mathcal{F}\left[g(t)\right](\omega) =$

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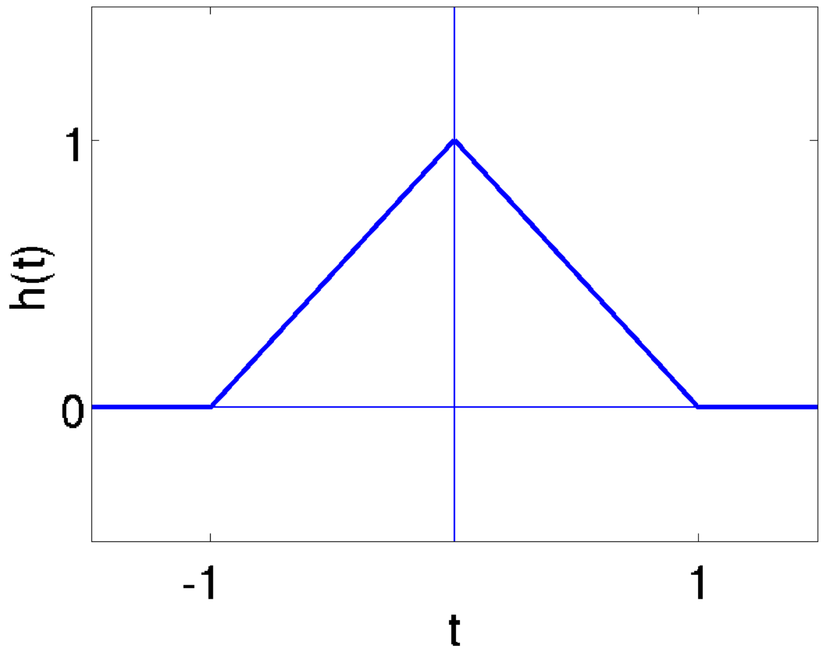
Part C (5 Marks)

Compute the Fourier transform of $h(t)$,



Hint: One way to find the transform of h starts by finding the transform of $\frac{dh}{dt}$.

$\mathcal{F}\left[h(t)\right](\omega) =$

Problem 7 (7 Marks)

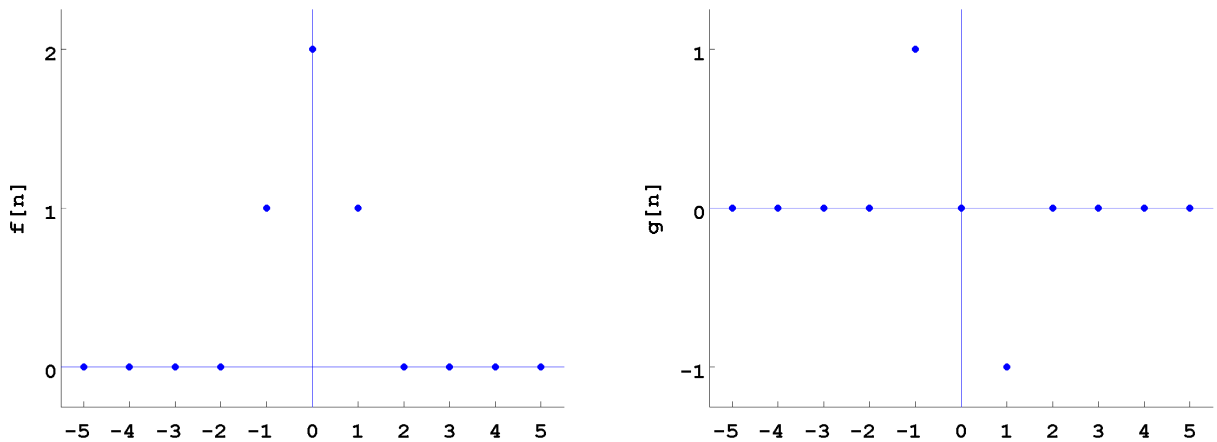
Using the definition of convolution, compute,

$$(q * q)(x), \quad \text{where,} \quad q(x) = x\,u(x)$$

$(q * q)(x) =$

Problem 8 (6 Marks)

Compute $(f \circledast g)[n]$, where the discrete-time signals $f[n]$ and $g[n]$ are given by,



Hint: You may leave your answer in terms of $g[n]$.

$(f \circledast g)[n] =$

Problem 9 (6 Marks)

Using the definition, compute the DTFT of $x[n] = 2^{-n}u[n+2]$

$\widehat{x}_{DT}(\omega) =$

Problem 10

Consider the discrete-time LTI described by the difference equation,

$$y[n] - y[n-1] = x[n] + x[n-1]$$

Part A (7 Marks)

Use the difference equation to find the impulse response, $h[n]$, with $h[-1] = 0$.

$$h[n] = \begin{cases} & \\ & \end{cases}$$

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Part B (1 Marks)

Is your answer to Part A right-sided, left-sided, neither, or both?

Part C (7 Marks)

Use the difference equation and your answer to Part B to find the system function, $H(z)$.

$H(z) =$

$ROC =$

Problem 11 (4 Marks)

Consider two discrete-time LTI circuits:

Circuit $\boldsymbol{\alpha}$ is causal, with system function,

$$F(z) = \frac{1}{z - \frac{2}{3}}$$

Circuit $\boldsymbol{\beta}$ has system function,

$$G(z) = \frac{z^2}{z^2 + \frac{3}{2}z - 1}; \qquad \frac{1}{2} < |z| < 2$$

Another circuit, $\boldsymbol{\gamma}$, is made by using the output of circuit $\boldsymbol{\alpha}$ as input for circuit $\boldsymbol{\beta}$.

What is the system function, $H(z)$, of circuit $\boldsymbol{\gamma}$?

$H(z) =$
$ROC =$

Problem 12

Part A (6 Marks)

Find the DFT of, $a[n] = [0, 1, 0, 1]$. Give your answer as a vector, fully-simplied.

$$\widehat{a}[k] \quad = \quad [\qquad , \qquad , \qquad , \qquad]$$

Part B (4 Marks)

Use your answer to Part A to find the DFT of $b[n] = [1, 0, 1, 0]$.

$$\widehat{b}[k] \quad = \quad [\qquad , \qquad , \qquad , \qquad]$$

Problem 13 (6 Marks)

Compute the DFT of $x[n] = \sin[\frac{\pi}{5}n]$.

Hint: Rewrite $x[n]$ *as complex exponentials.*

$$\widehat{x}[k] = \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

End of Exam

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