

**Math 267 Final Exam**

Apr 20, 2009

Duration: 150 minutes

Name: \_\_\_\_\_ Student Number: \_\_\_\_\_

Section: \_\_\_\_\_

**Do not open this test until instructed to do so!** This exam should have 17 pages, including this cover sheet. No textbooks, calculators, or other aids are allowed. One page of notes is allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam. **Circle your solutions! Reduce your answer as much as possible. Explain your work.** Relax. Use the back of the page if necessary.

**Read these UBC rules governing examinations:**

- (i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.
- (ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- (iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- (iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  - Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
  - Speaking or communicating with other candidates.
  - Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- (v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

<b>Problem</b>	<b>Out of</b>	<b>Score</b>
1	15	
2	15	
3	15	
4	15	
5	10	
6	15	
7	15	
<b>Total</b>	100	

**Problem 1 (15 points)**

Let  $f(t)$  be given by

$$f(t) = \begin{cases} t & \text{if } 0 \leq t < 1 \\ 2 - t & \text{if } 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Let  $f_e(t)$  be such that (i)  $f_e(t) = f(t)$  for all  $0 \leq t < 2$ , (ii)  $f_e(t)$  is periodic with period 2, and (iii)  $f_e(t)$  is **even**. Sketch the graph of  $f_e(t)$ .
- (b) Find the complex Fourier series of  $f_e(t)$ .
- (c) Find the real Fourier series of  $f_e(t)$ .



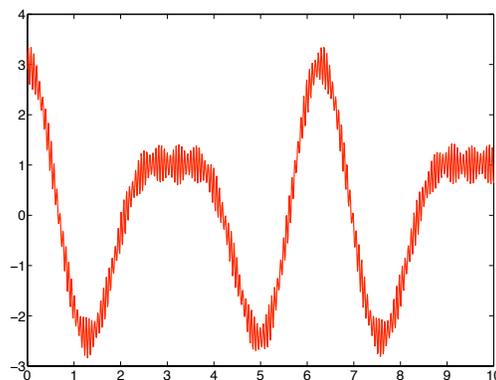
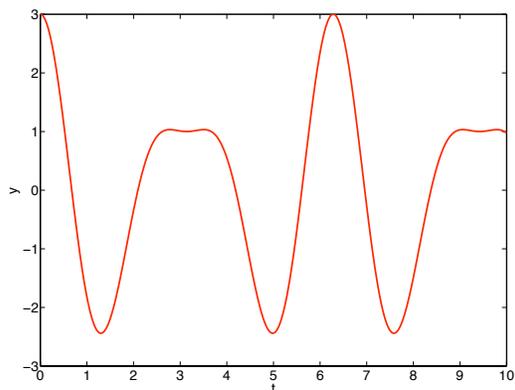
**Problem 2 (15 points)**

- (a) Find the inverse Fourier transform of  $\widehat{g}(\omega) = \text{sinc}(\omega - 3)$ .
- (b) Find the inverse Fourier transform of  $\widehat{h}(\omega) = \frac{i\omega}{(1+i\omega)(2+i\omega)}$ .
- (c) Find the Fourier transform of  $f(t) = te^{3t}u(-t + 2)$ .



**Problem 3 (15 points)**

Let  $f(t) = 2\cos(2t) + \cos(3t)$  be a signal of interest that was transmitted to us over a noisy channel. Suppose we received instead of  $f(t)$  a “contaminated” version given by  $x(t) = f(t) + n(t)$  where  $n(t) = 0.3\sin(101t) - 0.1\cos(117t)$ . (See figure for the graph of  $f(t)$  (left) and the graph of its contaminated version  $x(t)$  (right).



- (a) Calculate  $\hat{f}(\omega)$  and  $\hat{n}(\omega)$ .
- (b) Sketch the graph of  $\hat{x}(\omega)$ .
- (c) Now construct an LTI system (i.e., a filter) that will “denoise” the contaminated signal: the output of the system should be  $f(t)$  whenever its input is  $x(t)$ . Specify the transfer function of your system.



**Problem 4 (15 points)**

Find the solution  $u(x, t)$  to the wave equation

$$u_{tt}(x, t) = u_{xx}(x, t), \quad 0 < x < \pi, t > 0,$$

with the boundary conditions

$$u(0, t) = u(\pi, t) = 0$$

and the initial conditions

$$u(x, 0) = \sin(x) - \frac{1}{2} \sin(2x) \quad \text{and} \quad u_t(x, 0) = \sin(3x).$$



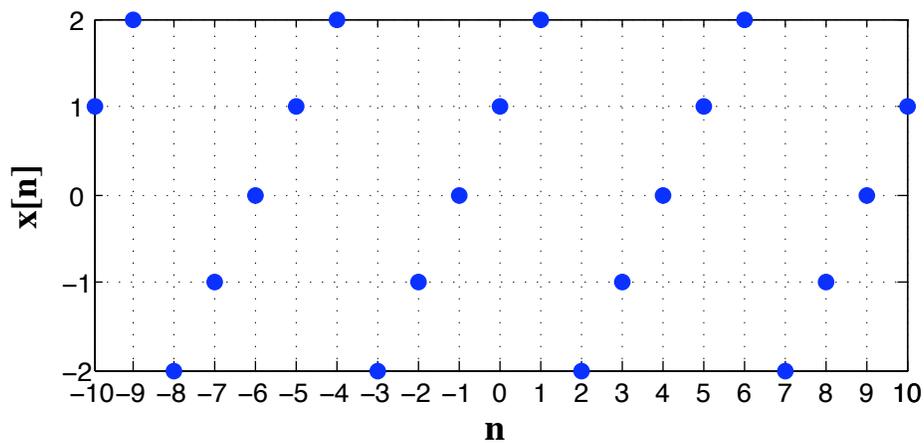
**Problem 5 (10 points)**

- (a) Find the discrete Fourier transform of the vector

$$v = [1, a, a^2, a^3, a^4, a^5, a^6],$$

where  $0 < a < 1$  is a parameter.

- (b) Find the discrete time Fourier series coefficients of the 5-periodic discrete time signal
- $x[n]$
- whose graph on the interval
- $-10 \leq n \leq 10$
- is given below.





**Problem 6 (15 points)** Determine the z-transform of the following signals.

(a)  $x[n] = \left(\frac{1}{3}\right)^n u(n)$ ,

(b)  $y[n] = \left(\frac{1}{3}\right)^n u(n - 3)$ ,

(c)  $y[n] = \left(\frac{1}{3}\right)^n u(-n - 1)$ .

(d) In each case above, indicate whether the discrete-time Fourier transform of the signal exists. If it does, find it. If not, explain.



**Problem 7 (15 points)**

Suppose that the expression for the z-transform of  $h[n]$  is given by

$$H(z) = \frac{z^2 + 3z}{(z - 1/2)(z - 2)(z - 4)}.$$

- (a) How many different regions of convergence could correspond to  $H(z)$ ? Specify each possible region of convergence.
- (b) If  $H(z)$  is the system function of a stable system, what is its region of convergence?
- (c) Find the impulse response of the system in part (b).
- (d) With the region of convergence from part (b), is the associated system causal? If not, can you choose a different region of convergence for  $H(z)$  so that the associated LTI system is causal? Justify your answer.





