Math 265 Final Exam

Monday, December 12, 2011

Duration: 3:30-6pm

Last Name: _____ First Name: _____ Student Number: _____

Do not open this test until instructed to do so!

This exam should have 20 pages, including this cover sheet. **The table of Laplace transforms is on Page 20**. You may tear out this page for convenience. You do not need to submit it with the exam.

This is a closed book exam; no textbooks, calculators, laptops, formula sheets or other aids are allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam.

Circle your solutions! Reduce your answer as much as possible. Explain your work. Relax. Use the extra pages if necessary.

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a UBCcard for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	**Out of**	**Score**
1	30	
2	24	
3	12	
4	12	
5	12	
6	10	
Total	100	

Problem 1 [30 points]:

Each of the following problems is worth 6 points.

1a) [6] Find the inverse Laplace transform of

$$F(s) = \frac{s-1}{s^2+s+1}.$$

1b) [6] Compute the convolution of $f(t) = t$ and $g(t) = t^2$.

1c) [6] Find the Laplace transform of the function

$$f(t) = \begin{cases} 2 & 0 \le t < 2, \\ -1 & 2 \le t < 3, \\ 0 & 3 \le t. \end{cases}$$

1d) [6] Place a condition on the value of k such that the solution of

$$3y'' + 2y' + ky = 0$$

exhibits oscillations.

1e) [6] Find all values of α such that the system

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} -2 & 0 \\ \alpha & -2 \end{bmatrix} \vec{x}$$

has a solution of the form $\vec{x}(t) = C_1\vec{v}_1 e^{-2t} + C_2(\vec{v}_1 t + \vec{\eta})e^{-2t}$ for *non-zero* vectors $\vec{v}_1$ and $\vec{\eta}$.

Problem 2 [24 points]:

Each of the following problems is worth 8 points.

2a) [8] Find the solution of

$$y' = \frac{y^2}{(x+2)^2} \,, \qquad y(0) = \frac{2}{3} \,.$$

2b) [8] Find the solution of

$$y' - \frac{4}{x}y = x^5 e^x \,, \qquad y(1) = 2 \,.$$

2c) [8] Find the general solution of

$$y'' - 4y = t.$$

Problem 3 [12 points]:

Consider the forced spring-mass system

$$y'' + y' + y = \cos(\omega t)$$

for a forcing frequency $\omega > 0$.

a) [6] Find the forced response (the steady-state solution) of the system (in terms of ω).

b) [3] Find the amplitude of the forced response.

c) [3] Determine the frequency ω, for which the forced response has the largest amplitude.

Extra page

Problem 4 [12 points]:

Find the solution of

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix} \vec{x}, \qquad \vec{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

Extra page

Problem 5 [12 points]:

Solve the initial-value problem

$$y'' - 7y' + 6y = \delta(t-2), \qquad y(0) = 1,\ y'(0) = 1.$$

Extra page

Problem 6 [10 points]:

For each of the following two-dimensional systems of differential equations identify the corresponding phase portrait given on the next page by writing clearly the appropriate letter (**A** - **I**) in the empty box. It is not necessary to solve the systems of differential equations. Each answer is worth 2 points, no partial credits will be given.

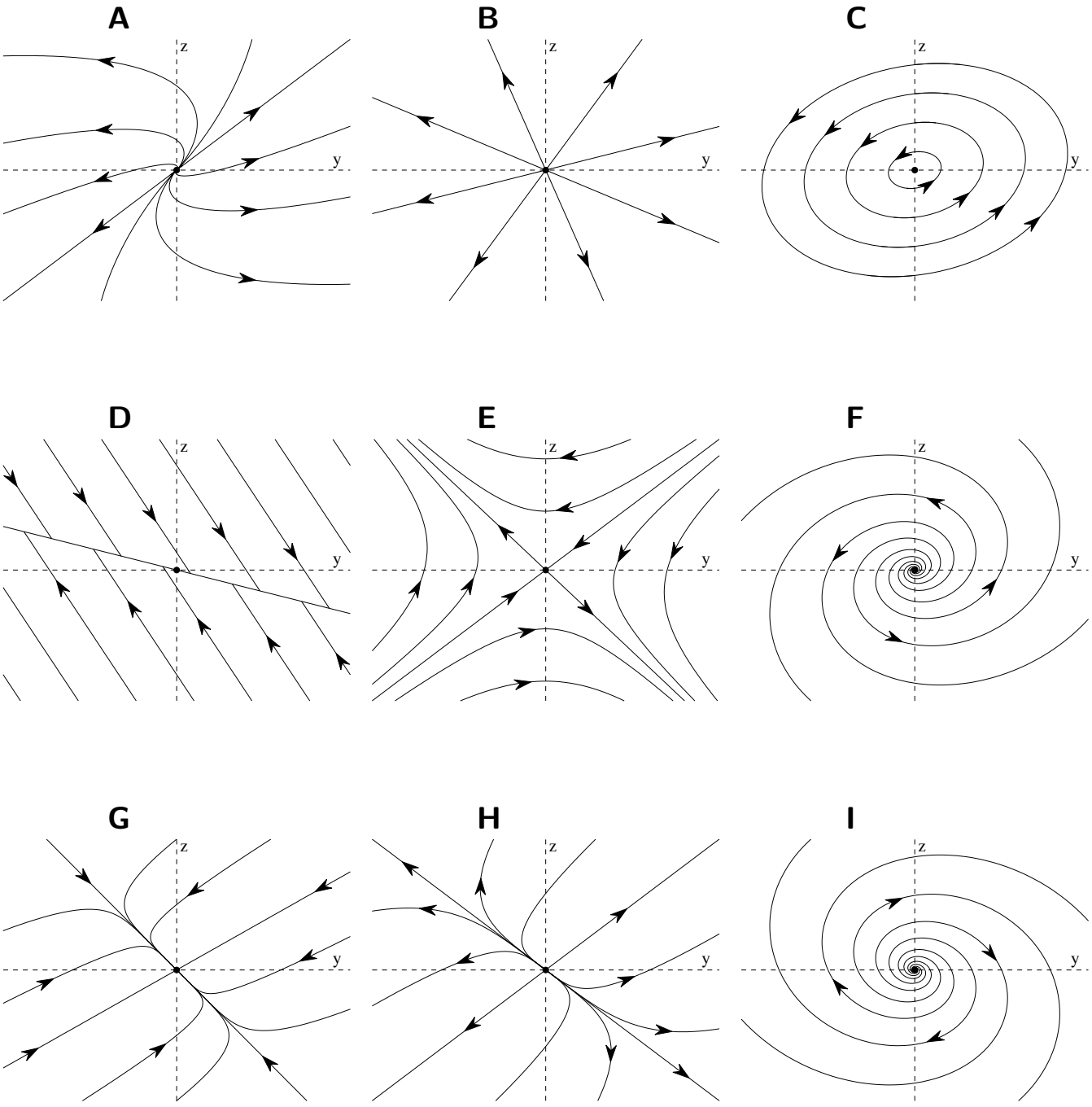
i) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ ☐

ii) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ ☐

iii) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ ☐

iv) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -2 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ ☐

v) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ ☐


A
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H
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Extra page

Extra page

Table of Elementary Laplace Transforms:

You may tear out this page for convenience. You do not need to submit it with the exam.

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
2.	e^{at}	$\frac{1}{s-a}$
3.	t^n, $n =$ positive integer	$\frac{n!}{s^{n+1}}$
4.	t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin(at)$	$\frac{a}{s^2+a^2}$
6.	$\cos(at)$	$\frac{s}{s^2+a^2}$
7.	$\sinh(at)$	$\frac{a}{s^2-a^2}$
8.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
9.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\frac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\frac{F(s/c)}{c}$
16.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - ... - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$