The University of British Columbia

Final Examination - December 2007

Mathematics 265

Section 102

Closed book examination Time: 2.5 hours

Last Name: _____ **First:** _____ **Signature** _____

Student Number _____

Special Instructions:

- Be sure that this examination has 11 pages. Write your name on top of each page.
- You are allowed to bring into the exam one $8\frac{1}{2} \times 11$ formula sheet filled on both sides. No calculators or any other aids are allowed.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		10
3		20
4		15
5		15
6		15
7		15
Total		100

[10] **1**. Find all solutions of $y' - 2xy^2 = 0$.

[10] **2**. Solve the initial value problem $xy' = x^3 - 2y$, $y(1) = 0$.

[20] **3**. Consider the initial value problem

$$y'' + ay' + by = 0, \qquad y(0) = 3,\ y'(0) = 5.$$

The differential equation has as a fundamental set of solutions $\{y_1(t), y_2(t)\}$, where $y_1(t) = e^{-t}$. The Wronskian of y_1 and y_2 is $W(t) = 4e^{2t}$.

(a) Solve for $y_2(t)$.

(b) Determine the values of the constants a and b.

(c) Solve the initial value problem.

Extra space (if needed)

[15] **4**. The homogeneous differential equation

$$t^2y'' - 2ty' + 2y = 0,$$

defined over the open interval $0.5 < t < 2$, has a non-trivial solution $y_1 = t^2$.

(a) Use reduction of order to find a second solution y_2.

(b) Show that y_1 and y_2 form a fundamental set of solutions.

(c) Find the particular solution that satisfies the initial conditions $y(1) = 3$ and $y'(1) = 4$.

Extra space (if needed)

[15] **5**. Solve the initial value problem

$$y'' + 2y' + 5y = f(t), \qquad y(0) = 1,\ y'(0) = -1,$$

where

$$f(t) = \begin{cases} 0 & \text{if } t < 1; \\ 1 & \text{if } 1 \leq t. \end{cases}$$

Extra space (if needed)

[15] **6**. Solve the initial value problem

$$x_1' = x_1 - x_2$$
$$x_2' = 5x_1 - 3x_2$$

with $x_1(0) = 1$, $x_2(0) = 3$. Describe the behaviour of the solution as $t \to \infty$.

[15] **7**. Find a fundamental matrix for the system of equations

$$\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 2 & 5 \end{pmatrix} \mathbf{x}.$$