The University of British Columbia

Final Examination - December 2009

Mathematics 263

Section 102

Closed book examination — Time: 2.5 hours

Last Name: ____________ **First:** ________ **Signature** ____________

Student Number ____________

Special Instructions:

- Be sure that this examination has 12 pages. Write your name at the top of each page.
- You are allowed to bring into the exam one $8\frac{1}{2} \times 11$ formula sheet filled on both sides. No calculators or any other aids are allowed.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;

  (b) speaking or communicating with other candidates; and

  (c) purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		20
2		15
3		20
4		20
5		10
6		15
Total		100

1. Suppose the function $T(x, y, z)$ describes the temperature at a point (x, y, z) in space, with $T(1, 1, 1) = 10$, and $\nabla T(1, 1, 1) = 2\hat{\imath} - \hat{\jmath} + \hat{k}$. Suppose also that the position at time t of a particle moving through space is $(\sqrt{1+t}, \cos t, e^t)$.

   (a) Compute the directional derivative of T at $(1, 1, 1)$, in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

   (b) At $(1, 1, 1)$, in what direction does the temperature decrease most rapidly?

(c) Compute the rate of change of temperature experienced by the particle at time $t = 0$.

(d) Write an equation for the tangent plane to the temperature level surface $T(x, y, z) = 10$ at $(1, 1, 1)$.

2. Use Lagrange multipliers to find the points on the surface $z = x^2 + 2y^2$ that are closest to the point $(0, 0, 2)$. (Hint: Minimize the distance squared rather than the distance.)

Extra space (if needed)

3. Let $\hat{F}(x, y, z) = \langle\, 0,\ xe^y,\ (z+1)e^z \,\rangle$.

   (a) Calculate the curl of $\hat{F}$.

   (b) Find a function $h(x, y, z)$ such that the vector field

   $$\hat{G}(x, y, z) = \langle\, h(x, y, z),\ xe^y,\ (z+1)e^z \,\rangle$$

   is conservative. Find a function $g(x, y, z)$ such that $\hat{G}(x, y, z) = \nabla g(x, y, z)$.

   (c) Evaluate the integral $\int_C \hat{G} \cdot d\hat{r}$, where the curve C is parametrized by $x(t) = t^2$, $y(t) = t^2$ and $z(t) = t^3$ for $0 \le t \le 1$.

   (d) Evaluate the integral $\int_C \hat{F} \cdot d\hat{r}$, where C is as in (c). (Hint: Use the results from (b) and (c).)

Extra space (if needed)

4. Let C be the closed curve oriented counterclockwise consisting of the line segment from $(0,0)$ to $(1,0)$, the line segment from $(1,0)$ to $(1,1)$ and the part of the parabola $y = x^2$ from $(1,1)$ to $(0,0)$. Find $\int_C \hat{F} \cdot d\hat{r}$ where $\hat{F}(x,y) = xy\,\hat{\imath} + x^2\,\hat{\jmath}$ by two methods:

   (a) By calculating the line integral directly.

   (b) By using Green's Theorem.

Extra space (if needed)

5. Use Stokes' Theorem to evaluate $\int_C \hat{F} \cdot d\hat{r}$, where C is the curve in which the plane $y = 1$ intersects the sphere $x^2 + y^2 + z^2 = 5$, oriented clockwise when viewed from the positive y-axis, and

$$\hat{F}(x, y, z) = \left(-y^2 + e^{x^2}\right) \hat{\imath} + \ln(y^2 + y)\, \hat{\jmath} + \left(x + \sqrt{z^2 + 1}\right) \hat{k}.$$

6. Let $\hat{F}(x, y, z) = \langle\, z \tan^{-1}(y^2),\ z^3 \ln(x^2+1),\ z\,\rangle$. Find the flux of $\hat{F}$ across the part of the paraboloid $x^2+y^2+z=2$ that lies above the plane $z=1$ and is oriented upwards.

Extra space (if needed)